Is Cryptocurrencies Extreme Returns-volumes Relationship Affected by COVID-19?

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Abstract: We explore the potential dependence between extreme return and volumes among different cryptocurrencies, using several different statistical models. Extreme return-volume dependence in Bitcoin, Ethereum, Ripple and Litecoin has been examined by copula methodology. We use EGARCH model for return series and GARCH model for volume series. For Bitcoin, by including the Covid-19 crisis, we have found that high volumes are not significantly dependent with high returns. Further, it has been found that (ETH, RIPPLE and LTC) may attract pessimistic investors due to insignificance of right tail dependence. For, Ethereum we have found evidence of low trading during the Covid-19 crisis due to significance of lower tail dependence coefficients. For, Litecoin extremely low volumes are more likely to coexist with extremely low and high returns before Covid-19 crisis. For, Bitcoin when include period of Covid-19 crisis, we found that trading increases for lower return which support the heterogenous investors with short sale constraintKeyword**:** Human Capital Index, Education, Health, Productivity of the future generation.

Keywords: Cryptocurrencies, EGARCH-Copula, returns-volumes, upper tail dependence, negative returns.

1. INTRODUCTION

The development of digital currencies since their introduction in 1990 has changed the structure and nature of financial transactions. The role of highly regularized financial institutions and regulators is under jeopardy as digital currencies evolve all around the world. Digital currencies provide alternative but decentralized systems of financial transactions without being part of the tight national and international monetary policies (Buchholz et al. 2012). This has led to an increase in the interest of the people since the introduction of Bitcoin by Nakamoto (2008). However, despite increased interest and media hype, the global cryptocurrency market is still smaller as compared to traditional fiat currencies. The total market capitalization of cryptocurrencies reached $600 billion in Dec 2017 before sliding down to $250 billion in July 2018. In terms of daily trading volume, cryptocurrency volumes are around $14 billion per day as compared to daily forex trade volume of $5 trillion. The spreads (bid-ask prices) of digital currencies are much higher (usually in dollars) as compared to spreads of forex trades (mostly in cents). Despite the increase in trading volumes and market capitalization, the cryptocurrency market is still considered as thinly traded market and is also very volatile.

The academic research to test the market efficiency and large disruptions in the movements of cryptocurrency prices has become known recently. Three main research questions have been answered in recent studies. (1) do cryptocurrency prices follow a random walk? (2) how new information about the changes in demand and supply of cryptocurrency is priced? and (3) does technical analysis based on stochastic martingale process help to predict future prices? The research has mainly remained divided on these issues, for example, Bariviera (2017), Nadarajah and Chu (2017) and Tiwari et al. (2018) concluded that cryptocurrency market is weak-form efficient and prices reflect all known information stored in historical prices. In contrast, Al-Yahyaee et al. (2018), Cheah et al. (2018) and Urguhart and McGroarty (2016) found no or weak evidence of the efficient pricing of cryptocurrency market in general. The main reason of these controversies lay down to very high level of uncertainty during the historical development of prices in cryptocurrency market. Recently, Khursheed et al. (2020) found that price movements with linear and nonlinear dependencies varies over time and thus the cryptocurrency market may remain efficient during certain times but not always. In the light of results obtained by previous literature, one issue that has not been studied extensively, is the relationship between return and trading volume in extreme scenarios. Identifying the time-varying relationship could be beneficial in understanding the rapid and extreme movements of prices in cryptocurrency market. Moreover, Covid-19 crisis affected global markets as well as cryptocurrency market. We therefore include the Covid-19 crisis period in our analysis.

Therefore, the primary aim of this paper is to examine extreme return-volume relationship in four most representative cryptocurrencies: Bitcoin, Ethereum, Litecoin and Ripple during the time period before and include Covid-19 crisis. The period of analysis is interesting because the prices of these four digital currencies reached to their all-time highs and lows during this time-period. Understanding the return–volume nexus can provide many useful signals for market participants to determine investment strategies or to rebalance their portfolios. The return and volume relationship has been studied in other financial markets such as equities,

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forex and fixed-income securities (Granger and Morgenstern 1963; Crouch 1970; Westerfield 1977; Tauchen and Pitts 1983; Rogalski 1978; Clark 1973). Theoretically, as investors revise their reservation prices based on the arrival of new information to the market, the changes in trading volume may serve as the proxy of new information and can directly be used to measure dispersion among investors (buyers and sellers). Epps and Epps (1976) showed a positive causal relation running from trading volume to absolute stock returns. Jain and Joh (1988) found strong contemporaneous relation between trading volume and returns by using hourly common stock trading volume and return on NYSE. Further, they also found lead-lag relationship between trading volume and returns lagged up to 4 hours.

Moreover, return-volume relation is asymmetric, i.e., higher for positive returns than for negative returns. Chen et al. (2001) studied the dynamic relation between trading volume, returns and volatility of stock indices of nine national markets. They found a positive dependence between trading volume and the absolute returns. Gunduz and Hatemi (2005) explored the causal relationship between stock prices and volume of Hungary, Czech Republic, Russia, Poland and Turkey stock markets and also found positive relationship. Floros and Vougas (2007) examined the relationship between trading volume and returns in Greek Stock Index Futures Market and found significant positive contemporaneous relationship between trading volume and returns in case of FTSE/ASE-20. Furthermore, Attari et al. (2012) and Kamath (2008) also reported the evidence on return-volume dependence. Cryptocurrency prices exhibit higher volatility than other financial assets at any point of time. This makes the case of cryptocurrencies to study high and low tail dependence of returns more interesting. Earlier literature has found that there is a strong correlation between volatility and nexus between trading volume and returns of securities. A positive correlation between return and volatility also gives rise to the concept of higher trading volumes during extreme volatility regimes. Rossi et al. (2013) supported the notion that return-volume tend to show relatively strong upper tail dependence. Similarly, Ning and Wirjanto (2009) found upper tail dependence in return and volume series of East Asian stock markets. On the other hand, Chen et al. (2001) showed that negative return in period t raises volatility in period *t+1* therefore the return would be lower in the periods of high volatility. Further, explanation can be seen from Naeem et al. (2014), that when volatility increases, risk increases and returns decrease. If we combine work of Rossi et al. (2013) and the fact mentioned in the paper by Chen et al. (2001) and Naeem et al. (2014), then one should expect positive dependence between low return and volumes as well.

Akbulaev and Salihova (2019) examined the relationship between cryptocurrencies prices and volume using VAR modelling approach and found the transaction volume change is negatively affected by the past thirteen-day values and the price change is affected by 1% of significance level. Zhang et al. (2018) explored the Return-Volume Relationship for Bitcoin Market. They found ant persistent behaviour for the return-volume in the Bitcoin markets. Balcilar et.al (2017) employ a non-parametric causality-inquantiles test to analyse the causal relation between trading volume and Bitcoin returns and volatility. Their results reveal that volume can predict returns – except in Bitcoin bear and bull market regimes. They suggested that this result highlights the importance of modelling nonlinearity and accounting for the tail behaviour when analysing causal relationships between Bitcoin returns and trading volume. Naeem et al. (2019) explore the time varying returns-volumes dependence of different cryptocurrencies but they did not include any event such as Covid-19 crisis in their analysis.

In this paper, we also consider negative return-volume relationship, in order to explore the upper tail dependence between the negative return and volume. That is the dependence between the lower tail of return and upper tail of volume. Furthermore, we consider return-volume dependence to analyse the difference between dependence parameter in both cases. Ning and Wirjanto (2009) used a copula approach to examine the extreme return-volume relationship in six emerging East-Asian equity markets. They used GARCH Copula approach. To the best of our knowledge, this is the first study, that considers cryptocurrencies to study return-volume relationship using EGARCH-Copula approach and Include the Covid-19 crisis.

Our goal in this paper is to explore the extreme dependence between return and volumes of four crypto currencies namely Bitcoin, Ethereum, Ripple and Litecoin. There are more than 300 different digital currencies available to buy however we selected the four most traded instruments in the market and these four currencies constitute more that 90% of the total market capitalization of the cryptocurrency market of the world. If cryptocurrencies returns are well described by the multivariate normal distribution, then the linear correlation is an appropriate dependence measure. However, in our case a simple exploratory and graphical analysis of both returns and volumes distributions suggest fat tails, heteroscedasticity, clustering and other non-Gaussian features of the distribution. Thus, a linear correlation might be deceptive. Therefore, alternative measures of dependence based on copula methods combined with EGARCH model are considered here. Copula approach is widely used in quantitative finance literature. Here we combine copula modelling with a univariate EGARCH model to model the returns of cryptocurrencies in order to properly calibrate a joint model for returns and volumes.

The remainder of this paper is organized as follows: section two introduces EGARCH methodology. Section three describes copula methodology. Section four reports empirical results and section five conclude with summary of our finding.

2. MATERIALS AND METHODS

ARCH model. ARCH models based on the variance of the error term at time *t* depends on the realized values of the squared error terms in previous time periods. The model is specified as:

$$y_t = u_t, \quad u_t \sim N, \tag{1}$$

$$h_t = \alpha_0 + \sum_{t=1}^{q} \alpha_j u_{t-i}^2. \tag{2}$$

This model is referred to as ARCH(q), where q refers to the order of the lagged squared returns included in the model. If we use ARCH (1) model it becomes:

$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2. \quad (3)$$

Since h_t is a conditional variance, its value must always be strictly positive; a negative variance at any point in time would be meaningless. To have positive conditional variance estimates, all of the coefficients in the conditional variance are usually required to be non-negative. Thus, coefficients must be satisfying $\alpha_0 > 0$ and $\alpha_1 \geq 0$.

GARCH model. Bollerslev (1986) and Taylor (1986) developed the GARCH (p,q) model . The model allows the conditional variance of variable to be dependent upon previous lags; first lag of the squared residual from the mean equation and present news about the volatility from the previous period which is as follows:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \quad (4)$$

In the literature most used and simple model is the GARCH (1,1) process, for which the conditional variance can be written as follows:

$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 h_{t-1}. \quad (5)$$

Under the hypothesis of covariance stationarity, the unconditional variance h_t can be found by taking the unconditional expectation of Equation 5.

We find that

$$h = \alpha_0 + \alpha_1 h + \beta_1 h. \quad (6)$$

Solving the equation (5), we have

$$h = \frac{\alpha_0}{1-\alpha_1-\beta_1}. \quad (7)$$

For this unconditional variance to exist, it must be the case that $\alpha_1 + \beta_1 < 1$ and for it to be positive, we require that $\alpha_{10} > 0$.

Exponential GARCH. Nelson (2006) proposed exponential GARCH (EGARCH) which has form of leverage effects in its equation. In the EGARCH model the specification for the conditional covariance is given by the following form:

$$log(h_t) = \alpha_0 + \sum_{j=1}^{q} \beta_j \, log(h_{t-j}) + \sum_{i=1}^{p} \alpha_i \left| \frac{u_{t=i}}{\sqrt{h_{t=i}}} \right| + \sum_{k=1}^{r} \gamma_k \frac{u_{t-k}}{\sqrt{h_{t-k}}}. \quad (8)$$

Two advantages stated in Brooks(2008) for the pure GARCH specification; by using log (h_t) even if the parameters are negative, will be positive and asymmetries are allowed for under the EGARCH formulation. In the equation γ_k represent leverage effects which accounts for the asymmetry of the model. While the basic GARCH model requires the restrictions the EGARCH model allows unrestricted estimation of the variance. If $\gamma_k < 0$ it indicates leverage effect exist and if $\gamma_k \neq 0$ impact is asymmetric. The meaning of leverage effect bad news increase volatility.

Applying process of GARCH models to return series, it is often found that GARCH residuals still tend to be heavy tailed. To accommodate this, rather than to use normal distribution the Student-t and GED distribution used to employ ARCH/GARCH type models.

2.1. Statistical Inference

Parameter estimation of GARCH and EGARCH model is commonly carried out by using the maximum likelihood method with normality assumption for ε_t. However, Kang et al. (2010) and Tang and Shieh (2006) mentioned that the residuals estimated from the GARCH type model frequently exhibits lepto-kurtosis and asymmetry. To overcome these problems the Student-t distribution has been considered for the innovations process. Given the random variable $\varepsilon_t \sim t_v(0,1,v)$ the log-likelihood function is defined as follows:

$$log(L;\theta) = T\left\{log\Gamma\left(\frac{v+1}{2}\right) - log\,\Gamma\left(\frac{v}{2}\right) - \frac{1}{2}log[\pi(v-2)]\right\} - \sum_{t=1}^{T}\left[log\,\sigma^2_t + (1+v)\,log\left(1 + \frac{\varepsilon^2_t}{\sigma^2_t(v-2)}\right)\right] \quad (9)$$

$$\varepsilon_t = {r_t}/{\sqrt{\sigma_t}} \quad (10)$$

MATLAB has been used to estimate the parameters of the GARCH and EGARCH models. then the standardized residuals are calculated as follows.

3. THE COPULA METHODOLOGY

Copula-based models provide a great deal of flexibility in modelling multivariate distributions. This allows the researcher to specify the models for the marginal distributions separately from the dependence structure (copula) that links them to form a joint distribution. From an inferential perspective the copula representation facilitates estimation of the model in stages, reducing the computational burden.

Several surveys of copula theory and applications have appeared in the literature to date: Nelsen (2007) and Joe (1997) are the most important books on copula theory, providing detailed introductions to copulas and dependence modelling, with an emphasis on statistical foundations. Kurowicka and Joe (2001) represents an up-to-date survey on copula and vine-copula applications Cherubini et al. (2004) present an introduction to copulas using methods from mathematical finance, Demarta and McNeil (2005) present an overview of copula methods for risk management. Patton (2008) presents a summary of applications of copulas to financial time series. Jondeau and Rockinger (2006) proposed a GARCH-Copula approach to measure the dependence structure of stock markets. It is well known that the analysis of dependence analysis, especially of extreme events, plays a crucial role in financial applications such as portfolio selection, Value-at-Risk, and international asset allocation.

A copula model is a way of constructing the joint distribution of a random vector $X = (X_1, \cdots, X_m)$. It is possible to show that there always exists an m-variate function ***C***: $[0, 1]^m \rightarrow [0, 1]$, such that

$$\boldsymbol{F}(x_1, \cdots, x_m) = C(F_1(x_1), \cdots, F_m(x_m)) \quad (11)$$

The copula function C is a cumulative distribution function (CDF) with uniform margins on [0, 1]: it binds together the univariate cumulative distribution functions F_1, F_2, and F_m to produce the m-variate CDF **F**. The three main properties are (1) $C(x_1, x_2, \cdots x_m)$ is increasing in component x_i; (2) $C(1, \cdots, 1, x_i, 1, \cdots, 1) = x_i$ for all $i = 1, \cdots, m, x_i \in [0,1]$;

For all $(\alpha_1, \ldots \alpha_m)$, $(b_1, \ldots b_m)$ ε $[0,1]^m$ with $a_i \leq b_i$ one has

$$\sum_{i_1=1}^{2} \cdots \sum_{i_m}^{2} (-1)^{i_1 + \cdots + B_m} C(x_{1i_1}, \cdots, x_{mi_m}) \geq 0$$

where $x_{j1} = a_j$ and $x_{j2} = b_j \; \forall \, j \in \{1, \cdots, m\}$.

For any continuous multivariate distribution, the copula representation is unique. If the marginal $F_1, \cdots, F_m$ are not all continuous it can be shown that the joint CDF still have a copula representation although this representation is not unique. In the continuous case one can take derivatives of both side of Equation (11), we get the density representation of F:

$$f(x_1, x_2, \cdots, x_m) = \frac{\partial^m F(x_1, \cdots, x_m)}{\partial x_1, \cdots, \partial x_m} = \frac{\partial^m C(F_1(x_1), \cdots, F_m(x_m))}{\partial F_1(x_1) \cdots \partial F_m(x_m)} \times$$

$$f_1(x_1) \times \cdots \times f_m(x_m) = c(F_1(x_1), \cdots, F_m(x_m)) \times \prod_{i=1}^{m} f_i(x_i), \quad (12)$$

where $c(u_1, \cdots, u_m)$ is the density of copula C, and $f_i(x_i)$ is the density of i-th margin. The joint use of GARCH and Copula models separates the temporal dependence, absorbed by the univariate GARCH structure, and the co-dependence among different variables, which is captured by the copula model.

3.1 Tail dependence and some bivariate copulas

In this paper, we use the copula approach to measure the tail dependence between the return and volume among four crypto currencies, so we keep focus on the two-dimensional case only.

3.1.1 Tail dependence

We can use the tail dependence coefficient to measure the concordance between the extreme events of different random variables. It is expressed in terms of a conditional probability that the asset *X* will incur a large loss (or gain), given that the asset *Y* also experiences a large loss (or gain). We consider two random variables *X* and *Y*, with joint continuous CDF ***F***, copula C, margins F_X and F_Y; the lower tail dependence and the upper tail dependence are defined as follows:

$$\lambda_L = \lim_{u \to 0^+} Pr(F_X(x) < u) \,|F_Y(y) < u) = \lim_{u \to 0^+} \frac{C(u,u)}{u} \quad (13)$$

$$\lambda_U = \lim_{u \to 1^-} Pr(F_X(x) > u) \,|F_Y(y) > u) = \lim_{u \to 1^-} \frac{1 - 2u + C(u,u)}{1 - u}. \quad (14)$$

Intuitively, if λ_L and λ_U exist and fall in (0, 1], *X* and *Y* show lower or upper tail dependence. On the other hand, if λ_L and λ_U are equal to 0, one can say that the two variables are independent in the tails, so extreme events seem to occur independently. We can describe different tail dependence behaviour by choosing the appropriate copula model.

3.1.2 Archimedean Copulas

Archimedean copulas are defined through their generator functions. Generally, if a function $\varphi : [0,1] \to [0, \infty]$ with the continuous derivative is decreasing and convex, it can be considered as a generator function of Archimedean copula. By definition an-dimensional Archimedean copula has the following expression: $C(u_1, u_2, \cdots, u_n) = \varphi^{-1}(\varphi(u_1) + \varphi(u_2) + \cdots + \varphi(u_n))$, different generator function creates different Archimedean copula. More details about generator function can be found in [36,37]. In our case the copula function is defined by:

$$C(u,v) = \varphi^{-1}(\varphi(u) + \varphi(v)) \quad if\ \varphi(u) + \varphi(v) \le \varphi(0), \quad (15)$$

where $\varphi(u)$ is a C^2 function with $\varphi(1) = 0,\ \varphi' < 0, \varphi'' > 0$.

Examples of Archimedean copulas include the following:

Clayton copula. The Clayton copula has the following form:

$$C(u, v; \rho) = max\left[(u^{-\rho} - v^{-\rho} - 1, 0)\right]^{-1/\rho},$$

$$\rho \in (-1, +\infty) \setminus \{0\} \quad (16)$$

where is the dependence parameter $\lambda_L = 2^{-1/\rho}$, $\lambda_U = 0$. When $\rho \to 0$, the margins tend to be independent, oppositely when $\rho \to \infty$, the margins tend to be strongly dependent. Clayton copula is asymmetric and it shows stronger low tail dependence. It can be proved that the components of a Gaussian copula are asymptotically independent.

Gumbel copula. The Gumbel copula is an asymmetric extreme value copula, which takes the following expression:

$$C(u, v; \rho) = exp\left[(-lnu)^{\rho} + (-lnv^{-\rho})\right]^{-1/\rho}$$

$$, \rho \in [1, \infty) \quad (17)$$

where ρ is a dependence parameter that describes different dependence behaviour, $\lambda_L = 0, \lambda_U = 2 - 2^{1/\rho}$. When $\rho \to \infty$ the margins show totally dependence, while $\rho = 1$ corresponds to independence case. Unlike the Clayton copula, Gumbel copula deals with upper tail dependence. If two margins perform simultaneous extreme upper tail values, the Gumbel copula should be an appropriate considerable choice.

Symmetrized Joe–Clayton copula. Joe (1997) constructed the copula by taking a particular Laplace transformation of Clayton's copula. The Joe–Clayton copula is:

$$C_{JC}(u, v|\tau^U, \tau^L) = 1 -$$

$$\left(1 - \{[1 - (1 - u)^k]^{-\gamma} + [1 - (1 - v)^k]^{-\gamma} - 1\}^{-1/\gamma}\right)^{1/k}, \quad (18)$$

where $k = \frac{1}{log_2(2 - \tau^U)}$, $\gamma = \frac{-1}{log_2(\tau^L)}$ and $\tau^i \in (0,1)$ are the measures of the upper- and lower-tail dependencies respectively. Patton (2006) modified Joe–Clayton (JC) copula for which the density is as follows.

$$C_{JC}(u, v|\tau^U, \tau^L) = 1 -$$

$$\left(1 - \{[1 - (1 - u)^k]^{-\gamma} + [1 - (1 - v)^k]^{-\gamma} - 1\}^{-1/\gamma}\right)^{1/k}, \quad (19)$$

The SJC copula is symmetric when $\tau^U = \tau^L$ and asymmetric otherwise.

3.2 Copula Parameters Estimation

Most of the methods for copula parameter estimation are related to Maximum Likelihood procedures. The standard ML method which estimates both marginal parameters and copula parameters simultaneously is also named one step method. Mashal and Naldi (2002) noted that this method is computational costly, and when the data sets are not sufficiently large, the ML estimators seem to be ineffective. The inference function for margins method (IMF) is based on the work of Joe and Xu (1996). The estimation procedure is split in two steps; first one estimates the parameters of the marginal distributions. In the second step one tries to estimate of the copula parameters, conditionally on the values of estimates obtained at the first step. This approach offers computational convenience, although it may be sensitive to the choice of marginal distributions form. A poor estimator of the copula parameter might be a consequence of an inappropriate marginal distribution. There is also an alternative, two steps method, named Canonical Maximum Likelihood (CML). Unlike IMF method, in the CML approach the transformation is done by using empirical CDF function to obtain uniform margins, which are used in copula parameters estimation. Given two time series $\{X\}_{\tau}^{T}=1$ and $\{Y\}_{\tau}^{T}=1$, let Ω be the parameter space, $\alpha_x \varepsilon \Omega$, α_y, $\varepsilon\ \Omega$ denote marginal parameters for *X* and *Y*, while $\theta\ \varepsilon\ \Omega$ denotes copula parameters. From Equation (12), the log maximum likelihood function can be obtained as:

$$(\alpha_x, \alpha_y, \theta; X, Y) = \sum_{t=1}^{T} lnc\left(F_X(x_t;\alpha_x), F_Y(y_t;\alpha_y);\theta\right) + \sum_{t=1}^{T}\left(lnf_X(x_t;\alpha_x) + lnf_y(y_t,\alpha_y)\right) \quad (20)$$

Here we sketch the necessary inferential steps.

Step 1. Estimating parameters of the marginal distributions, α_x and α_y.

$$\hat{\alpha}_x = \underset{\alpha_x}{argmax} \sum_{t=1}^{T} lnf_X(x_t, \alpha_x) \quad (21)$$

$$\hat{\alpha}_y = \underset{\alpha_y}{arg\,max} \sum_{t=1}^{T} lnf_Y(y_t, \alpha_y) \quad (22)$$

Step 2. Estimating the copula parameters by using the estimator $\hat{\alpha}_x$ and $\hat{\alpha}_y$ obtained in step 1.

$$\hat{\theta} = \underset{\theta}{argmax} \sum_{t=1}^{T} lnc\left(F_X(x_t;\hat{\alpha}_x), F_Y(y_t;\hat{\alpha}_y);\theta\right) \quad (23)$$

The copula parameters were estimated by employing the maximum likelihood method described in Equation 23. For the IMF estimation, a MATLAB copula toolbox written by Patton (2008) has been used.

4. EMPIRICAL STUDIES AND ANALYSIS

4.1 Primary Data Analysis

In empirical studies, we choose daily prices and corresponding trading volume series of four crypto currencies, Bitcoin (BTC), Ethereum (ETH), Ripple and Litecoin (LTC) out of the top five largest-capped cryptocurrencies (Figure 1). There two time period associated with data one ranges from 24 August, 2016 to 30 January 2020 and other ranges from 26 February 2018 to 15/10/2022.

The first time period is before the Covid-19 crisis and the second time period include the data associated with cryptocurrencies during the Covid-19 crisis. Ethereum (ETH) is the second cryptocurrency in terms of market capitalization. For an exhaustive analysis of the cryptocurrency market, please refer to Feng et al. (2018).

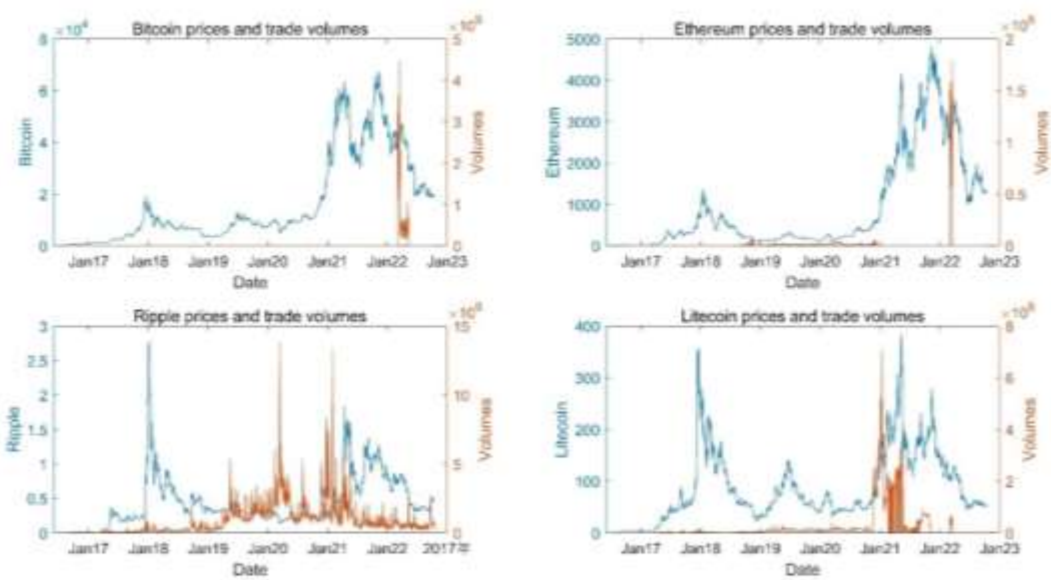

Fig. (1). Daily prices and trading volumes fluctuations.

Our focus is on the four currencies placed in the top five in terms of market capitalization for three reasons. On the one hand, they represent more than 50% of the cryptocurrency market capitalization. On the other hand, most of investors are attracted by these consecrated cryptocurrencies and may construct a portfolio based on these assets. In addition, we want to keep simple and clear our empirical exercise based on copula function. Fig. **1** illustrate the relative price movements of each crypto currency. We take the daily log returns defined as $R_t = 100x\log\left(P_t/P_{t-1}\right)$ which can be seen in Fig. **2**.

The preliminary descriptive statistics four major cryptocurrencies (BTC, ETH, RIPPLE and LTC) are presented in Table 1. As depicted in Table **1**, the average returns for all cryptocurrencies are positive before the Covid-19 crisis but the becomes close to zero or even negative including the Covid-19 crisis period for all cryptocurrencies. The distributions of the four major cryptocurrencies price returns for both time periods are non-normal, asymmetric and fat-tail distribution. Hodrick and Prescott (1997) filter have been used to remove the trend from the log-volume series. As shown in Table **1**, the kurtosis of each adjusted volume is greater than 3 and the skewness is not zero, which both suggest that presence of fat-tails and leptokurtosis. Moreover, graphs of autocorrelations for both return and adjusted volume have significant autocorrelations which need to be addressed by using appropriate models.

4.2 Marginal Distribution Models

We continue the investigation with the identification of the marginal models for both time periods. The order for the ARMA part has been chosen, after careful inspection of ACF and PACF of both return and adjusted volume series. We estimate the ARMA-EGARCH and ARMA-GARCH models which is robust to tail behaviour and volatility clustering existed in the return and adjusted volume series and results presented in the Table **2** and Table **3**.

The mean equation has few statistically significant parameters in all four models while the variance equation showed statistically significant parameters for all the four models. Thus, while past returns are not instantaneously and rapidly embodied current returns, hence there is no such evidence of autoregressive components presented in all the currencies. The variance equation shows evidence of significant autoregressive components in all models. Thus, one-day lagged shocks could affect their current conditional volatility. Moreover, the persistence parameter appears largest for Bitcoin and least in the case of Ripple, suggesting high level of persistence in all series (LTC, ETH and RIPPLE) followed by Bitcoin. The appropriateness and reliability of the GARCH marginal model were confirmed using a number of diagnostic tests such as ARCH-LM test.

Further, residuals and squared residuals series do not possess significant autocorrelation for both return and volume series as it can be seen in Figs. **3** and **4**. All the estimations have been performed by using the MATLAB toolbox ARIMA. The test shows that residuals are approximately i.i.d. series, therefore copula approach can be applied to the residuals after getting student-*t* CDF from the residuals.

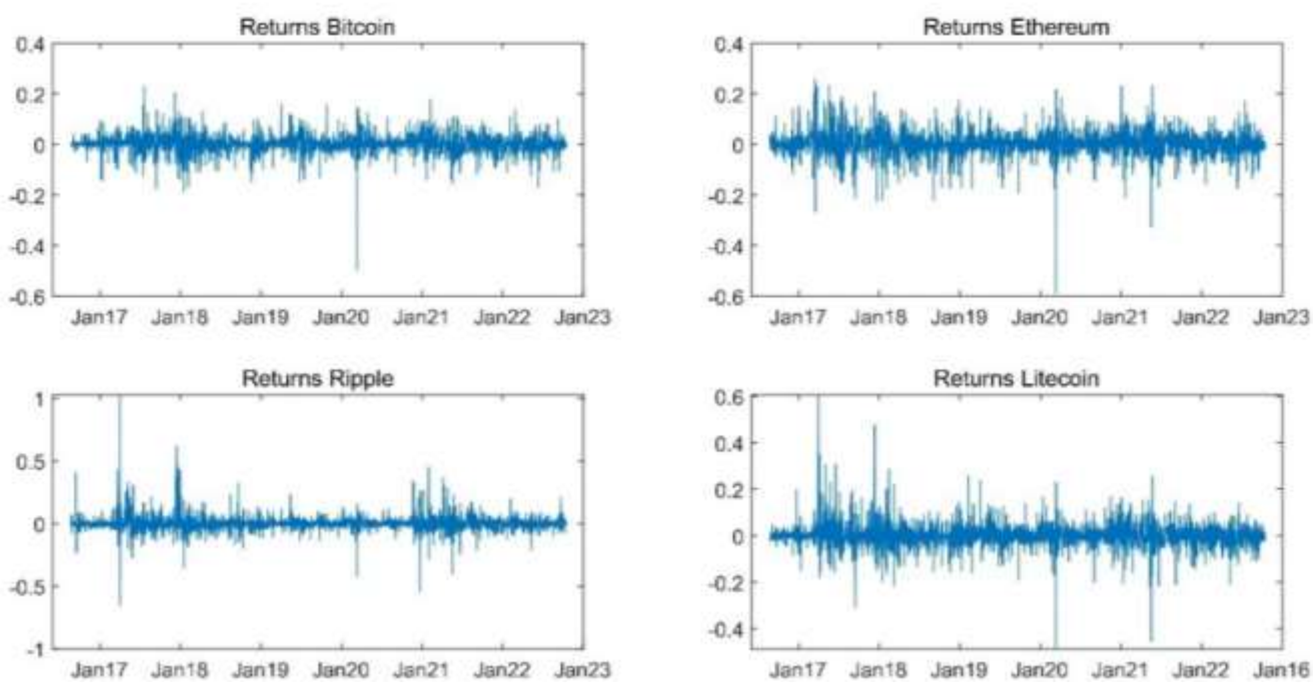

Fig. (2). Daily returns of each cryptocurrency.

Table 1. Descriptive statistics of the sample data.

Descriptive Statistics	Bitcoin	Ethereum	Ripple	Litecoin
Pre-Corona-crisis				
observations	1255	1255	1255	1255
mean	0.0022	0.0022	0.0029	0.0022
std	0.0409	0.0557	0.0772	0.0623
max	0.2276	0.2585	1.0279	0.6069
min	-0.1869	-0.2666	-0.6529	-0.3079
skewness	-0.0724	0.1462	2.8098	1.7704
kurtosis	6.5718	6.1665	39.5866	16.6658
ARCH-LM	108.07	125.66	156.49	80.05
Adjusted Volume				
mean	12.4342	14.4497	19.4521	14.7735
std	1.1247	1.4780	2.1654	2.3932
skewness	-0.0895	-0.3387	-1.7495	-1.3386
kurtosis	1.9604	2.4274	5.6594	4.9531
ARCH-LM	2225	2225	2213	2225
Include-Corona-crisis				
Mean	0.0004	0.0002	-0.0003	-0.0008
Std	0.0387	0.0511	0.0585	0.0536
Max	0.1774	0.2307	0.4489	0.2581
Min	-0.4972	-0.5896	-0.5410	-0.4867
Skewness	-1.2710	-1.1446	0.0915	-0.8031
kurtosis	20.6766	15.7574	16.6345	11.8648
ARCH-LM	24.12	41.84	94.92	74.52

Note: ARCH-LM is the Engel's LM test for heteroscedasticity, conducted using 20 lags. The test result show that the series are serially correlated as the test static exceed the critical value=3.841.

Table 2. Parameter estimation of GARCH and EGARCH model before Corona-crisis.

	Bitcoin		Ethereum		Ripple		Litecoin	
	Return							
parameters	value	SE	value	SE	value	SE	value	SE
C	0.0025*	0.0014	-0.0001*	0.0002	-0.0047	0.0014	-0.0011*	0.0009
AR (1)	0.1424*	0.4250	-0.1264*	0.3279	-0.0030*	0.1294	0.3940	0.1567
SAR (2)	-0.8146	0.1382	0.7586	0.1614	-0.8260	0.0426	-0.8713	0.0711
MA (1)	-0.1948*	0.4202	0.0147*	0.3288	-0.1907*	0.1245	-0.4969	0.1470
SMA (2)	0.8317	0.1330	-0.7335	0.1767	0.8202	0.0465	0.8807	0.0693
Dof	2.07056	0.05625	2.5469	0.2849	2.0691	0.0652	2.3277	0.1813
K	0.0146*	0.0213	-0.2197	0.0883	-0.0847	0.0398	-0.0694*	0.0389
β	1	0.0048	0.9576	0.0155	0.9745	0.3310	0.9845	0.0072

α	0.6235	0.2492	0.3506	0.0864	-0.0144	0.7204	0.3518	0.0937
Leverage	0.0896*	0.0633	0.0013*	0.0308	-0.0144*	0.0720	-0.0105*	0.0337
Dof	2.07056	0.0562	2.5469	0.2849	2.0691	0.0652	2.3277	0.1813
ARCH test	3.8174	-	6.0442	-	0.0352		0.0383	-
	Volumes							
parameters	value	SE	value	SE	value	SE	value	SE
C	0.0420*	0.0753	0.4531	0.1012	0.0508	0.0118	0.0559	0.0262
AR (1)	-0.3936	0.0291	-0.3719	0.0275	0.5767	0.0443	0.9932	0.0027
SAR (2)	0.9967	0.0044	0.9784	0.0050	0.9940	0.0020	0.4539	0.0905
MA (1)	0.9933	0.0034	0.9798	0.0054	0.0870*	0.0486	-0.3901	0.02670
SMA (2)	-0.3882	0.0280	-0.3805	0.0278	-0.8929	0.0166	-0.6564	0.0753
Dof	4.1794	0.36312	6.1952	0.8694	6.2382	1.1997	6.8568	1.2334
K	0.0227	0.0052	0.0004*	0.0004	0.0000*	0.0003	0.0004*	0.0003
β	0.5905	0.0652	0.9353	0.0117	0.9695	0.0071	0.9522	0.0094
α	0.2897	0.0702	0.0647	0.0131	0.0302	0.0075	0.0457	0.0104
Dof	4.1794	0.36312	6.1952	0.8694	6.2382	1.1997	6.8568	1.2334
ARCH test	0.0354	-	2.3034	-	28.0997		4.7514	-

Notes: Table 2 reports the estimated parameters for EGARCH and GARCH models for returns and adjusted volumes respectively, together with standard errors. * indicates that the parameters are not significant at both 5% and 10% significance level. Dof denotes degree of freedom.

Table 3. Parameter estimation of GARCH and EGARCH model include Corona-crisis.

	Bitcoin		**Ethereum**		**Ripple**		**Litecoin**	
	Returns							
parameters	**value**	**SE**	**value**	**SE**	**value**	**SE**	**Value**	**SE**
C	0.0017*	0.0014	0.0016*	0.0019	-0.0007*	0.0008	-0.0006*	0.0019
AR (1)	-0.3373*	0.2795	-0.1658*	0.4331	0.2436*	0.2025	-0.2551*	0.2770
SAR (2)	-0.6600	0.1147	-0.6238	0.2889	0.0277*	0.6239	-0.6802*	0.5180
MA (1)	0.2671*	0.2840	0.0756*	0.4341	-0.3979	0.1990	0.1516*	0.2797
SMA (2)	0.6745	0.1151	0.6645	0.2634	0.0085*	0.6102	0.6697*	0.5311
Dof	2.8417	0.2358	3.4423	0.3472	2.5800	0.2068	3.4822	0.3427
K	-0.0692*	0.0448	-0.3678	0.1333	-0.2314	0.0682	-0.2647	0.1001
β	0.9891	0.0068	0.9379	0.0224	0.9581	0.0117	0.9548	0.0170
α	0.1898	0.0322	0.1963	0.0415	0.3564	0.0623	0.1831	0.0366
Leverage	0.0071*	0.0164	-0.0173*	0.0205	0.0277*	0.0273	-0.0020*	0.0198
Dof	2.8417	0.2358	3.4423	0.3472	2.5800	0.2068	3.4822	0.3427
ARCH test	0.0562	-	0.3510	-	0.9786	-	0.8497	-
	Volumes							
parameters	value	SE	value	SE	value	SE	value	SE
C	0.2100	0.0625	0.0014*	0.0009	0.2738	0.0853	-0.0021*	0.0107
AR (1)	-0.2181	0.0153	0.9857	0.0061	0.9821	0.0046	1	0.0017

SAR (2)	0.9846	0.0039	0.9932	0.0042	0.2381	0.0969	0.5904	0.0876
MA (1)	0.9911	0.0025	-0.4985	0.0219	-0.2785	0.0235	-0.4248	0.0232
SMA (2)	-0.2027	0.1493	-0.9850	0.0053	-0.4567	0.0891	-0.6948	0.0714
Dof	2.0001	0.0000	3.0158	0.2177	5.341	0.7943	2.9788	0.1931
K	71.2138	0.0043	0.0401	0.0085	0.0005*	0.0004	0.0207	0.0038
β	0.9344	0.04461	0.5283	0.0580	0.9555	0.0103	0.6188	0.0296
α	0.0656*	4.2499	0.4051	0.0945	0.0436	0.0104	0.3812	0.0818
Dof	2.0001	0.0000	3.0158	0.2177	5.341	0.7943	2.9788	0.1931
ARCH test	0.0471	-	349.56	-	5.0954	-	0.8985	-

Notes: Table **3** reports the estimated parameters for EGARCH and GARCH models for returns and adjusted volumes respectively, together with standard errors. * indicates that the parameters are not significant at both 5% and 10% significance level.

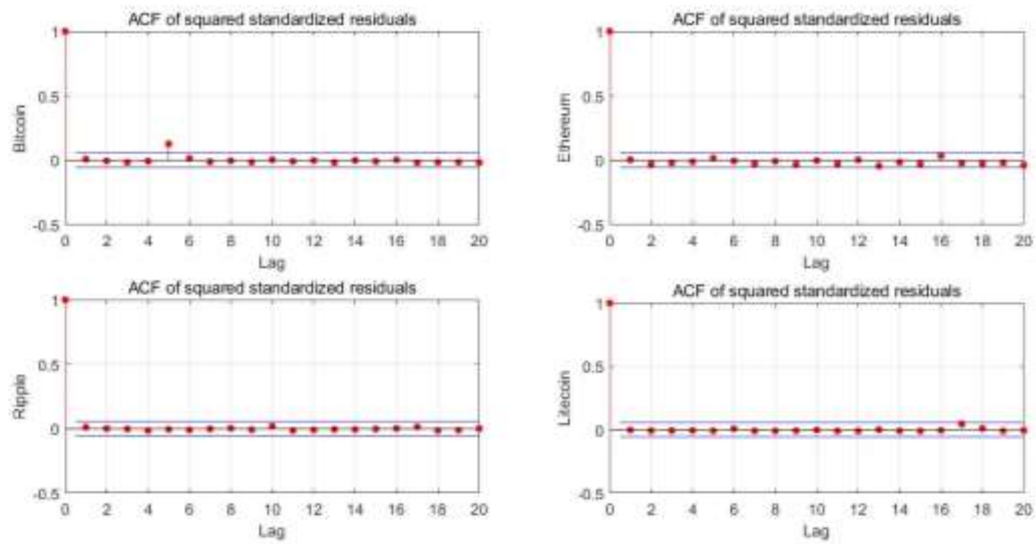

Fig. (3). ACF of squared standardized residuals of Returns.

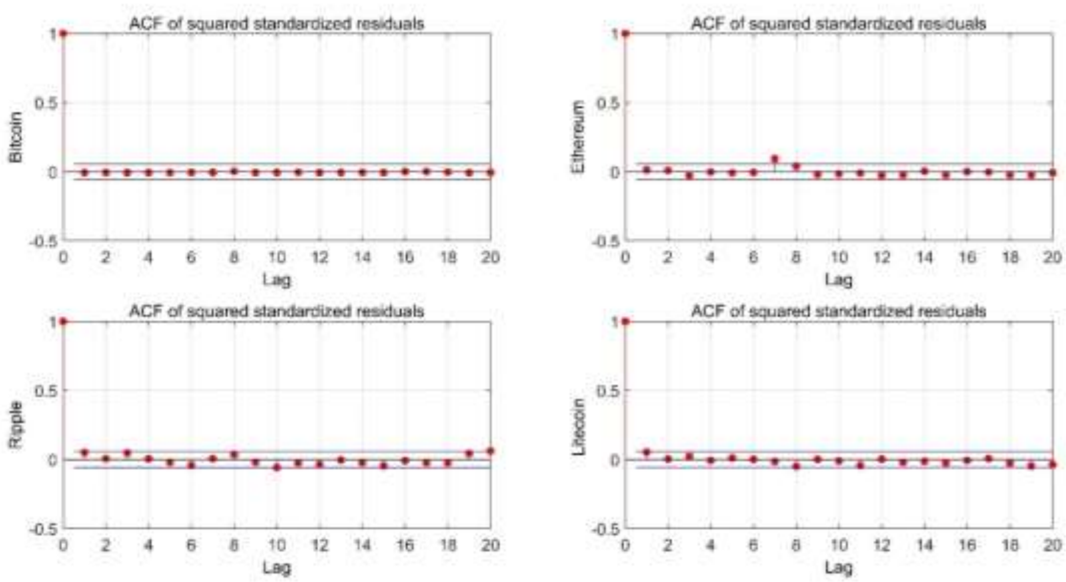

Fig. (4). ACF of squared standardized residuals of volumes.

4.3 Copula Parameter Estimation

We are interested in the dependence structure between the cryptocurrencies returns and trading volumes. Our main goal being to explore the extreme dependence between return and volumes. We employed 3 copulas in our analysis, namely Clayton, Gumbel and SJC copulas. These copulas measure asymmetric dependence between returns and volumes. The asymmetric copulas are able to capture upper and lower tail dependence coefficients. Based upon the Log Likelihood function the parameter estimates for each copula, belong to data before-Covid-19 crisis and including the Covid-19 crisis, have been reported in Tables **4** and **5** respectively. Further, summary of upper and lower tail dependence coefficients has been reported in Table **6**.

Table 4. Copula estimates of return-volume dependence before-corona-crisis.

	Positive returns				**Negative returns**			
	Bitcoin	**Ethereum**	**Ripple**	**Litecoin**	**Bitcoin**	**Ethereum**	**Ripple**	**Litecoin**
	Clayton Copula							
Parameter	0.4104 (0.0606)	0.3963 (0.0632)	0.6596 (0.0692)	0.4809 (0.0667)	0.0026* (0.0793)	0.0001* (0.0600)	0.0028* (0.0738)	0.0001* (0.0804)
	Gumbel Copula							
P	1.4242 (0.0418)	1.4138 (0.0434)	1.6662 (0.0549)	1.4962 (0.0486)	1.100 (0.0471)	1.100 (0.0407)	1.100 (0.0398)	1.100 (0.0409)
	SJC Copula							
Upper tail	0.0000* (0.0178)	0.0000* (0.0000)	0.0618* (0.8577)	0.0012* (0.0097)	0.0023 (0.0000)	0.0000* (0.0000)	0.0052* (0.0000)	0.0000* (0.0000)
Lower tail	0.4473* (1.1775)	0.4364* (0.7891)	0.5641* (7.7817)	0.4930 (0.0281)	0.1389* (0.8602)	0.0000* (0.4776)	0.1392* (0.3354)	0.0000* (0.0127)

Notes: Table 4 reports the estimates of parameters of 3 copulas for each pair of return and volume, together with standard errors (in parentheses). *indicates the parameters are **not significant** at 5% significance level.

Table 5. **Copula estimates of return-volume dependence including Corona crisis.**

	Positive return				**Negative returns**			
	Bitcoin	Ethereum	Ripple	Litecoin	Bitcoin	Ethereum	Ripple	Litecoin
	Clayton Copula							
P	0.3832 (0.0511)	0.2945 (0.0517)	0.4976 (0.0529)	0.2074 (0.0509)	0.0001* (0.0354)	0.0001* (0.0503)	0.0025* (0.0645)	0.0001* (0.0425)
AIC	-65.9591	-39.3052	-105.148	-19.4705	-0.0288	0.0584	1.4505	0.0391
	Gumbel Copula							
P	1.3719 (0.0358)	1.2934 (0.0325)	1.5595 (0.0419)	1.2379 (0.0313)	1.1000 (0.0348)	1.1000 (0.0370)	1.1000 (0.0367)	1.1000 (0.0338)
AIC	-191.061	-129.563	-328.885	-95.1459	75.8773	95.3271	102.735	75.2050
	SJC Copula							
Upper tail	0.0037 (0.0704)	0.0000 (0.0000)	0.0000 (0.0666)	0.0000 (0.0126)	0.0000 (0.0716)	0.0000 (0.0706)	0.0000 (0.0707)	0.0000* (0.0000)
Lower tail	0.3971 (0.0553)	0.3275 (0.0504)	0.5362 0.0480	0.2705 (0.000)	0.0000 (0.0328)	0.1704 (0.0468)	0.0000 (0.0472)	0.0000* (0.000)
LL	-96.7123	-65.5422	-171.528	-48.9788	5.6854	10.6784	9.1694	6.1882

Note: Table **3** reports the estimates of parameters of six copulas for each pair of return and volume, together with standard errors (in parentheses). *indicates the parameters are **not significant** at 5% significance level.

Table 6. Upper and lower tail dependence coefficients.

	Bitcoin		**Ethereum**		**Ripple**		**Litecoin**	
	Pre-corona-crisis							
	positive	**negative**	**positive**	**negative**	**positive**	**negative**	**positive**	**negative**
Upper tail= θ_1								
λ_{ub}^{G}	**0.3731**	0.1221	**0.3673**	**0.1222**	**0.4841**	**0.1222**	**0.4108**	**0.1222**
λ_{ub}^{SJc}	**0.0000**	0.0023	**0.0000**	**0.0000**	**0.0618**	**0.0052**	**0.0012**	**0.0000**
Lower tail= θ_2								
λ_{Lb}^{C}	**0.1847**	**0.0000**	**0.1739**	**0.0000**	**0.3496**	**0.0000**	0.2366	**0.0000**
λ_{Lb}^{SJc}	**0.4473**	**0.1389**	**0.4364**	**0.0000**	**0.5641**	**0.1392**	0.4930	**0.0000**
	Corona-crisis							
	positive	negative	positive	negative	positive	negative	positive	negative
Upper tail= θ_3								
λ_{U}^{G}	0.3426	0.1222	0.2910	0.1221	0.4403	0.1221	0.2495	**0.1222**
λ_{U}^{SJc}	0.0037	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
Lower tail= θ_4								
λ_{L}^{C}	0.1638	0.0000	0.0950	0.0000	0.2483	0.0000	0.0354	**0.0000**
λ_{L}^{SJc}	0.3971	0.0000	0.3275	0.1704	0.5362	0.0000	0.2705	**0.0000**

We can see from the Table 6 that in most cases lower tail dependence for both positive and negative return-volume dependence is significant during the market stress. Specifically, low volumes are not associated with high and low returns before Covid-19 crisis on the other hand by including Covid-19 crisis low volumes are significantly dependent with high and low returns except in Litecoin. During crisis investor expect prices to fall therefore doesn't take part in trading and optimist investor wait for situation to get better when the trend changes, even if the trend is upward price ticks. Which means that during the market stress most of the investor still wanted to keep the cryptocurrencies and new investor afraid to invest at upward price ticks resulted decrease in trading volumes.

We have noticed from table 6 that in negative-volume relationship for Bitcoin upper tail dependence coefficients are significant for both time period (before Covid-19 crisis and include Covid-19 crisis). Which means that extremely high volumes are significantly dependent with extremely low returns.

Further, we have observed that upper tail dependence coefficients for both negative and positive return volume relationship before Covid-19 crisis are not significant for ETH, RIPPLE and LTC. Which clearly indicate that neither extremely high volumes are significantly dependent with extremely low returns nor extremely high volumes associated with extremely high returns in non-crisis environments. Moreover, our analysis show that when time period of Covid-19 crisis included, BTC, ETH and RIPPLE are very sensitive for extremely upward or downward price ticks but for LTC it is only sensitive to upward price ticks. This information supports the view of heterogenous investors with short buy constraints. The plausible reason is that focusing on heterogeneous investors, bullish investors did not initially participate in the market under normal market conditions because of short-buy constraints. When markets rise, bearish investors bail out of the market and bullish investors become the marginal supporting buyers. More signals and hidden information regarding bullish investors are revealed and learned. After digesting the newly released hidden information, fully rational, risk-neutral arbitrageurs re-enter the market to short the position, which results in an increase in market participants and trading volumes. We can see from the table 6 that in negative return-volume relationship for Litecoin both upper and lower tail dependence coefficients are not significant for both time period (before Covid-19 crisis and include Covid-19 crisis). Which means that extremely high or low volumes are not significantly dependent with extremely low or high returns. These results indicate that there is no dependence have been identified for both time period in negative return-volume relationship. Leverage effect is referred to an asymmetric negative correlation between return and the volatility. In our study we found not found any evidence of leverage effect.

CONCLUSIONS

We have analyzed the extreme tail dependence structure between return-volume and negative return-volume for two time periods (i.e., time period before the Covid-19 crisis and time period which include Covid-19 crisis). Our analysis was based on modeling dependence structure via Copula methodology. We have filtered out margins with EGARCH and GARCH models for returns and volumes respectively.

We have used three tail dependent copulas namely Clayton, Gumbel and SJC copula. Left and Right tail dependence coefficients for both negative and positive return-volume relationship time period which include Covid-19 crisis are significant except in Litecoin. Which illustrate that, low and high volumes significantly dependent with both low and high returns but are not significantly in case of Litecoin (time period which include the Covid-19 crisis). Further, our results for four crypto currencies have been affected by the Covid-19 crisis. Moreover, for Litecoin there is no evidence of negative return-volume relationship during the both time period (before Covid-19 crisis and include Covid-19 crisis).

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