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Assessing the Hedge Potential of Brent Crude Oil against the Currency of Oil-Exporting and Oil-Importing Countries: Application of C-Vine Model

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Abstract: This study scrutinizes the hedge capabilities of Brent Crude Oil against the currency of oil exporting countries and importing countries. The research is relies on the C-Vine model. The findings affirm a negative dependence among Brent Crude Oil and all currencies through the full sample period, and mainly during the pandemic of COVID-19 and the Russian invasion of Ukraine. Outcome of analysis support the prominence of Brent Crude Oil as a hedge and as a safe haven against the fluctuations of main currencies. Particularly, examination results highlight that increases in Brent oil prices in countries that are net oil exporters and importers are associated with the depreciation of that country's currency against the dollar. Specifically, study results mean that a rise in the oil price is related to a dollar appreciation and vice versa. Against the backdrop of this crisis, examination results reveal that there is exceptionally a positive correlation between Brent and the Russian ruble during the war between Russia and Ukraine. The Investigative conclusions support that oil is a weak hedge and a weak safe haven against the Russian ruble.

Keywords: Crude Oil; Exchange rate; Hedging; C-Vine Copula; Crisis.

1. INTRODUCTION AND LITERATURE REVIEW

The crude oil represents the most traded commodity in the world (Regnier, 2007). Oil prices are the leading economic variable driving the global economy and the world oil prices have seen different variation. By consequence, the oil market becomes very volatile and risky. In addition, Crude oil prices are determined by global supply and demand. As the US dollar is the main currency for international crude oil trade, the relationship between commodities and exchange rates has recently been an interesting to examine.

On the one hand, participants in the oil market, particularly traders, producers and consumers, are not only concerned by variations in the oil prices. On the other hand, they are interested in the relationship that exists between energy commodity prices and currencies (Amin, et al., 2022, Nandelenga, et al., 2020, Beckman, et al., 2020, and Zorgati, 2023).

The relationship between oil price and the dollar exchange rate is examined by Golub, (1983). Results show that the exchange rates of oil-exporting countries appreciate when oil price rise, and it depreciates the exchange rates of oil-importing countries. He explains by this why the oil will be sold at a high price and oil-importing countries have to pay more and then the value of oil-exporters currencies will be appreciated against US dollar.

In the same context and by applying copula models Reboredo, 2011 studies the dependence structure among crude oil prices. Findings show significant symmetric higher and minor tail dependence among variables. The principal conclusion of this study notes that oil prices are linked with the similar strength through bull and bear markets. These findings support the hypothesis that the oil market constitutes a vast unified reservoir.

However, by using the structural vector autoregression model and calculating the impulse responses, Basher, et al. (2012) examine the dynamic relationship among oil prices, exchange rates and emerging market stock prices. Their results approve that positive shocks to oil prices have a tendency to depress stock prices and US dollar exchange rates in the short term. In addition, increases in stock prices seem to increase oil prices.

Likewise, Reboredo, and Rivera-Castroet (2013) explore the oil price-exchange rate relationship across various timeframes. Their study demonstrates that, prior to the crisis, there was no dependence between oil prices and exchange rates for both crude oil prices and a range of currencies. However, following the onset of the crisis, evidence of contagion and negative dependence emerged. Moreover, it was observed that during the crisis period, oil prices had a leading effect on exchange rates and vice versa.

With the objective of examining the transmission of returns and volatility between oil prices and the exchange rate, Salish and Mobolaji, (2013) look for detecting the spillover effects, by employing a VAR-GARCH model. Using the results derived from the VAR-GARCH model, the study investigates the optimal asset weights for holding oil and foreign exchange, while also calculating the hedging ratios in the presence of oil-related risks. The research uncovers ro-

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bust structural breaks that correspond with both the global financial crisis and foreign exchange crisis. Furthermore, the study confirms a bidirectional transmission of returns and spillover effects among considered variables. Notably, the results highlight the efficiency of hedging strategies concerning oil and foreign exchange markets in such context. Consequently, incorporating oil into a diversified portfolio of foreign exchange assets can enhance the portfolio's risk-adjusted return performance.

Considering the Asian case by observing and analyzing the impact of oil price shocks on its currency rates, Nusair and Olson, (2019) fall back on the quantile regression analysis and account for structural shifts and asymmetry. The findings reveal that positive and negative oil price fluctuations have asymmetrical consequences on exchange rate returns, with varying significance, magnitude, and direction across the distribution of exchange rate returns. The influence of oil price shocks is also contingent upon market situations. Particularly, in bullish markets for domestic currencies, escalating oil prices lead to additional appreciation of currencies. Conversely, in bearish markets for domestic currencies, rising (falling) oil prices result in further currency depreciation. Consequently, exchange rates reply distinctively to oil price fluctuations during extreme bullish or bearish exchange market situations.

Using an alternative analytical methodology, Kumar (2019) study the asymmetric impact of oil price on exchange rate. Particularly he employs the Hiemstra and Jones nonlinear Granger causality, and novel NARDL tests. As results, it is affirmed that significant reaction nonlinear causality runs among oil price and exchange rate. As well, preceding month positive and negative shocks in oil prices have positive significant impact on exchange rate. The positive preceding month shock has more marked impact than the negative shock.

Moreover, to observe the link among oil prices and exchange rates. Beckmann, et al. (2020) start with theoretical transmission channels. They confirm that results vary substantially according to sample, country choice and empirical technique. Furthermore, they ascertain that strong links among exchange rates and oil prices are regularly noted over the long-run. Also, they affirm either exchange rates or oil prices are a possibly suitable forecaster of the other variable in the short-run, but the effects are powerfully time-varying.

In Addition, the multiscale spillovers and nonlinear causalities among the crude oil futures market and the stock markets of the United States, Canada, China, Russia, and Venezuela before and through the COVID-19 pandemic is examined by Ali et al., (2022). They rely on the wavelet coherency method. They find robust co-movement among considered variables, particularly in the early period of the COVID-19 epidemic, at high frequency. Besides, they note positive co-movements at low frequency during the total period. These results propose that the bearish trend of stock markets is related to a descending movement in oil prices. Using the wavelet-based Granger-causality approach, findings show that the oil and stock indices have less co-movement on a reduced scale but bigger movement on a higher scale through wholly areas. In this context, they conclude that investors should hold further oil futures than stock shares in their portfolios for wholly times. Their conclusion is that oil instruments are considered as important for hedging through normal periods and act as safe-haven assets during abnormal periods.

Furthermore, the connectedness among the exchange rate, domestic crude oil price is detailed and investigated by Chuanwang et al.(2022). Applying this study to the case of China, they refer to the MS-VAR model. Results show that, in this context, oil price is powerfully impacted by the international crude oil market. Placebo test results confirm that the positive impact of the China’s exchange rate on the International Energy Exchange crude oil futures price may be transferred to China's crude oil spot market.

Additionally, Sokhanvar and Houri (2023) Scrutinize the effect of commodity price shocks linked to the war in Ukraine on the Canadian dollar, euro, and Japanese yen. By means of four-hour price data for wheat, crude oil, and natural gas and EUR/CAD and CAD/JPY, the study founded on the quantile autoregressive distributed lag model recommends a long-run association among higher commodity prices and appreciation of the Canadian dollar against the euro and the yen. Besides, the dynamic simulated quantile autoregressive distributed lag model illustrates a positive impact of commodity price shocks on the value of the Canadian dollar against the euro and the yen. The driving forces in the depreciation of the euro and the yen are raised gas prices and raised wheat prices, respectively.

Recently, Zorgati (2023) probes the risk spillover effect among the currencies of importing and exporting oil countries and the oil price. Findings confirm that oil price has a stronger spillover impact in the case of oil exporting countries and the lower spillover effect in the case of oil importing countries. The study is maintained by the application of a set of double-long memories. Then, a multivariate GARCH type model is used to examine the dynamic conditional correlations. Besides, the Gumbel copula is adopted to define the nonlinear structure of dependence and to evaluate the optimal portfolio. The conditional Value-at-Risk is considered as a risk measure. Results demonstrate that a long-run dependence and asymmetry of bidirectional risk spillover between oil price and exchange rate, and approve that the risk spillover intensity is different between the former and the latter. In particular, it’s confirmed that oil price has the heaviest spillover effect in the case of oil exporting countries and the weakest spillover effect in the case of oil importing countries.

Similarly, Chatziantoniou, et al.2023 delve into the contagion dynamics among different types of oil price shocks and exchange rates. Oil price shocks are considered as permanent net transmitters of shocks within the system. It is found that the oil shock net spillovers composed most of the net connectedness values in the majority of countries through the pre-pandemic period. They confirm that oil exporters and oil importers were all net receivers of shocks. However, through the pandemic era, there were significant distinctions among the cluster of the nations.

Likewise, Kyophilavong et al., 2023 assess the cross-spectral coherence and co-movement among the monthly return series of West Texas Intermediate oil price and exchange rate of Thai Baht against American dollar from 1986 to 2019. To make estimation, they refer to the quantile cross spectral (coherency) approach and time frequency wavelets. Findings show negative spillover effects of studied variables. They confirm that such results prove that oil market present systemic risk to the foreign exchange market. In the context of the wavelet analysis, they affirm the absence of movements at high frequency.

In this study, we examine the dependence between crude oil and currencies and try to verify whether oil is still the least risky choice compared to the foreign exchange market. In this paper, C-Vine copulas are applied to study portfolio strategies during the healthy and political crisis for the entire period 2015–2022. The COVID-19 health crisis and the geopolitical crisis triggered by the invasion of Ukraine by Russia were two periods of crisis.

After presenting the literature review section, the rest of the paper is organized as follows. The section 2 covers the methodological framework. The Section 3 provides the data and preliminary analysis. The section 4 discusses the empirical results. Finally, the section 5 draws the conclusion.

2. METHODOLOGY

Copulas are introduced by (Sklar, 1959) updated by (Genest and MacKay 1986), forming a multivariate distribution function. In particular, a d-dimensional copula is the distribution function of a random vector U whose components are uniformly distributed. That is:

for every random vector $u = (u_1, \ldots, u_d) \in (0,1)^d$,

$$C(U_1 \leq u_1, \ldots, U_d) = P(U_1 \leq u_1, \ldots, U_d \leq u_d) \quad (1)$$

For a random vector $X = (X_1, X_2, \ldots, X_d) \in \mathbb{R}^d$ with distribution function

$F(x_1, \ldots, x_d) = P(X_1 \leq x_1, \ldots X_d \leq x_d)$ and continuous marginal distribution function $F_i(x) = P(X_d \leq x)$ for all i=1,…,d

There exists a unique d-dimensional copula function C $\in [0,1]^d$ such that

$$\boldsymbol{F(x_1, x_2, \ldots, x_d) = C(F_1(x_1), F_2(x_2), \ldots F_d(x_d))} \quad (2)$$

where $(x_1, x_2, \ldots, x_d) \in \mathbb{R}^d$

From the previous equation, we obtain the copula formula as follow:

$$C(u_1, \ldots, u_d) = F\left(F_1^{-1}(u_1), \ldots, F_d^{-1}(u_d)\right) \quad (1)$$

Where we denote F_i^{-1} : the inverse marginal distribution functions and $u_i = F_i(x_i)$ for all i=1,…,d.

Multivariate copulas are implemented for dimensions $d \geqslant 3$ and are restricted to non-tail correlated Gaussian and symmetric Student-t in more complex copulas and asymmetric tail-dependent functions. A d-dimensional distribution is decomposed from the density into a series of connected bivariate copula trees constructed as blocks called regular vines (R-vines).

Vines are used to designate the graphical representation of the pair copula constructions (PCC) proposed by (Bedford and Cooke., 2001, Bedford, et al., 2002). Aas, et al., 2009 have showed that R-Vines are decomposed in two: C-vines (Canonical Vine) and D-vines (Drawable Vine). Dependence modeling using vine copulas offers a greater flexibility and permits the modeling of complex dependency patterns for high-dimensional distributions.

V is a vine on d variables if:

$$V = (T_1, \ldots, T_{d-1})$$

T_1 is a tree with nodes $N(T_1) = \{1, 2, \ldots, d\}$ and edges $E(T_1)$.

For $\ell > 1$,T_ℓ is a tree with nodes $N(T_\ell) = E(T_{\ell-1})$

For $i = 2, \ldots, d-1$, T_i is a tree with nodes $N_i = E_{i-1}$ and a set of edges E_i

V is called a regular vine on d elements if we add a third condition to the two previous ones:

For $i = 2, \ldots, d-1$, if $a = \{a_1, a_2\}$ and $b = \{b_1, b_2\}$ are nodes of T_i linked by an edge, then exactly one of the a_i equals one of the b_i.

The d-dimensional density is decomposed to the equation of the C-Vine given by:

$$f_C(x_1, \ldots, x_d) = \prod_{k=1}^{d} f(x_k) \prod_{j=1}^{d-1} \prod_{i=1}^{d-j} C_{j,j+i|1,\ldots,j-1}\left[F(x_j|x_1, \ldots, x_{j-1}), F(x_{j+i}|x_1, \ldots, x_{j-1})\right] \quad (2)$$

Where j denotes the trees, i the edges that connect these trees, and $C_{j,j+i|1,\ldots,j-1}$ is a bivariate copula density. The tree T_j has a unique node linked through edges to all the other $d-j$ nodes.

f_k: the marginal densities with $k = 1, \ldots, d$

The conditional distribution functions of the x_i is given by (Joe, 1997):

$$\boldsymbol{F_{i|j}(x_i, x_j) = \frac{\partial c_{i,j}(F_i(x_i), F_j(x_j))}{\partial F_j(x_j)}} \quad (5)$$

For example, the 5-dimensional C-vine density decomposition is given by:

$$\begin{aligned} f(x_1,x_2,x_3,x_4,x_5) = {} & c_{1,2}(F_1(x_1),F_2(x_2)).c_{1,3}(F_1(x_1),F_3(x_3)).c_{1,4}(F_1(x_1),F_4(x_4)) \\ & .c_{1,5}(F_1(x_1),F_5(x_5)).c_{2,3|1}(F_{2|1}(x_2|x_1),F_{3|1}(x_3|x_1)).c_{2,4|1}(F_{2|1}(x_2|x_1),F_{4|1}(x_4|x_1)) \\ & .c_{2,5|1}(F_{2|1}(x_2|x_1),F_{5|1}(x_5|x_1)).c_{3,4|1,2}(F_{3|1,2}(x_3|x_1,x_2),F_{4|1,2}(x_4|x_1,x_2)) \\ & .c_{3,5|1,2}(F_{3|1,2}(x_3|x_1,x_2),F_{5|1,2}(x_5|x_1,x_2)).c_{4,5|1,2,3}(F_{4|1,2,3}(x_4|x_1,x_2,x_3),F_{5|1,2,3}(x_5|x_1,x_2,x_3)) \\ & .f_1(x_1).f_2(x_2).f_3(x_3).f_4(x_4) \end{aligned} \quad (6)$$

(Aas, et al., 2009) propose two inference algorithms to estimate C- vine log-likelihoods. C- vine copula models are typically fitted sequentially by proceeding iteratively tree by tree and thus only involving bivariate estimation for each individual pair copula term (Czado, et al., 2012). Therefore, estimation of the parameters of each pair-copula can be carried out using inversion of Kendall's τ or MLE.

(Czado, et al., 2012) proposed a sequence estimation method for vine ligatures. We summarize as follows:

First estimate the parameters of the unconditional pair copulas in the first tree, then the parameter used to estimate the conditional pair copulas in the second tree, and also used to estimate the two conditional variable pair copulas in the third tree. Repeat this process until all parameters of the paired copula are estimated.

Table 1. Descriptive Statistics of Returns Series.

	Mean	Maximum	Minimum	Std.Deviation	Skewness	Kurtosis
Brent	0.0002699	0.41202	-0.643699	0.033953	-2.766159	88.2145
EUR	0.0001014	0.02259	-0.026006	0.004985	-0.026543	5.24938
CAD	8.255793e-05	0.02392	-0.028697	0.004875	-0.004375	4.93313
RUB	6.544084e-06	0.25884	-0.145388	0.015695	3.590823	74.0029
CNY	7.026998e-05	0.01816	-0.014285	0.002465	0.458218	8.82791

As a final step, use these sequential estimates as a starting value to calculate the maximum likelihood estimate (MLE) of the Vine copula.

Assume that we observe d variables at T time points.

Let $x_i = (x_{i,1}, \dots, x_{i,T})$;$i = 1, \dots, d$

Then the log-likelihood for C-vine is given by:

$$lnf(x) = \sum_{j=1}^{d-1}\sum_{i=1}^{d-j}\sum_{t=1}^{T} ln\left(c_{j,j+i|1,\dots,(j-1)}\left(F(x_{j,t}|x_{1,t},\dots,x_{j-1,t}),F(x_{i+j,t}|x_{1,t},\dots,x_{j-1,t})\right)\right) \quad (3)$$

$\theta_{j,i}$ is the set of parameters of the copula density $c_{j,j+i|1,\dots,(j-1)}$.

Similarly, the log-likelihood for D-vine is given as:

$$lnf(x) = \sum_{j=1}^{d-1}\sum_{i=1}^{d-j}\sum_{t=1}^{T} ln\left(c_{i,j+i|1,\dots,(i+j-1)}\left(F(x_{i,t}|x_{i+1,t},\dots,x_{i+j-1,t}),F(x_{i+j,t}|x_{i+1,t},\dots,x_{i+j-1,t})\right)\right)$$

(10)

$\theta_{j,i}$ is the set of parameters of the copula density $c_{i,j+i|1,\dots,(i+j-1)}$.

The conditional distribution functions are computed using (Joe, 1997) :

$$F(x|v) = \frac{\partial c_{x,v_j|v_{-j}}(F(x|v_{-j}),F(v_j|v_{-j}))}{\partial F(v_j|v_{-j})}$$

v is a d-dimensional vector

v_j is an arbitrarily chosen component of v

v_{-j} denotes the v -vector excluding the componentv_j .

If v is univariate, and x and v are uniformly distributed on [0,1], then $F(x|v) = \frac{\partial c_{x,v}(x,v,\theta)}{\partial v}$

Where θ is the set of copula parameters.

The procedures for the C-vine and D-vine differ in how $F(X_j|X_1, X_2, \dots, X_{j-1})$ is computed:

For the canonical vine,$F(X_j|X_1, X_2, \dots, X_{j-1})$) is given by:

$$\frac{\partial c_{j,j-1[1,\dots,j-2}(F(x_j|x_1,\dots,x_{j-2}),F(x_{j-1}|x_1,\dots,x_{j-2})}{\partial F(x_{j-1}|x_1,\dots,x_{j-2})} \quad (4)$$

For the D-vine ,$F(X_j|X_1, X_2, \dots, X_{j-1})$ is given by:

$$\frac{\partial c_{j,1|2,\dots,j-1}(F(x_j|x_2,\dots,x_{j-1}),F(x_1|x_2,\dots,x_{j-1})}{\partial F(x_1|x_2,\dots,x_{j-1})} \quad (12)$$

4. EMPIRICAL METHODOLOGIES

4.1. Data Description

Countries selected in terms of oil production are: Russia (RUB), United European (EUR), Canada (CAD) and China (CNY). We defined exchange rates as the amount of local currency to one US Dollar. The real crude oil prices are defined as the spot price per barrel denominated in US dollar: BRENT. In our data set, all variables start at 02/01/2015 and end in 05/12/2022. Table **1** offers the summary of the data set.

To verify that our returns series are indeed stationary, we use the unit root test Augmented Dickey-Fuller Test (ADF) and the stationarity test Kwiatkowski–Phillips–Schmidt–Shin (KPSS). The results show that the p-values of all the variables are smaller than 5% which means that we reject the null hypothesis of non-stationarity. Thus, all series are stationary.

4.2. Normalitiy Test

There are graphical and statistical methods to test the normality of data. We use histograms, QQ-plots and the Jarque Bera Test to check if the variables are normally distributed or not.

It is possible to visualize the shape of the distribution of the data to be analyzed by plotting it as a histogram and comparing the shape of the normal distribution. As we can see in Fig. (**1**), the histogram of our daily return's series with the curve of the normal distribution shows that the unconditional return distribution (red line) differs from the normal one. In fact, it is sharper than the normal distribution (blue line).

Graphical QQ-Plots allow comparison of sample quantiles with quantiles of normally distributed samples with the same sample mean and the same variance. If the latter follows a normal distribution, these points must be confused with the first bisector of the plan. We find the QQ plot easier to interpret than the histogram because it allows us to see exactly where the data deviates from normal. Fig. (**2**) shows that all returns are symmetric, with tails that deviate significantly from the line (red), indicating that the distribution is more marginal than normal.

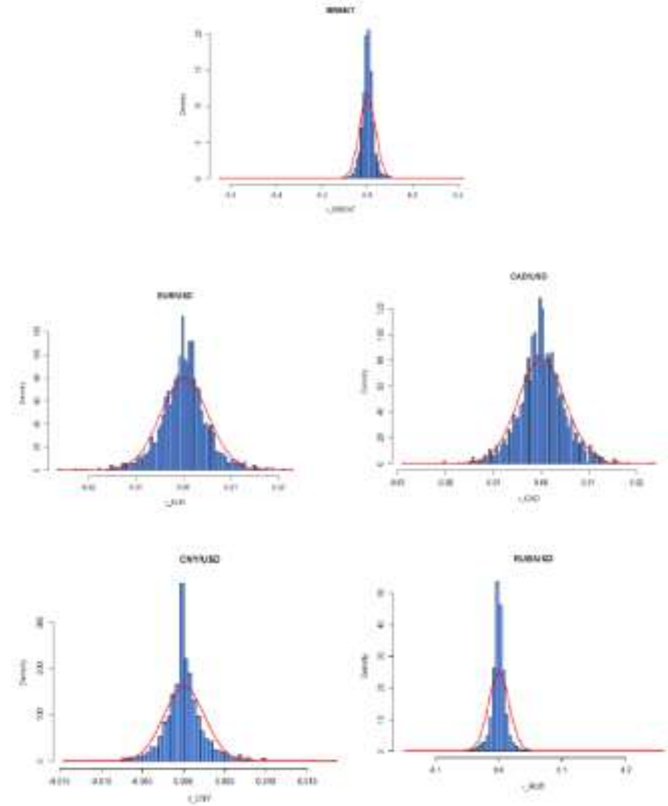

Fig. (1). Histograms with the normal curve of Log returns.

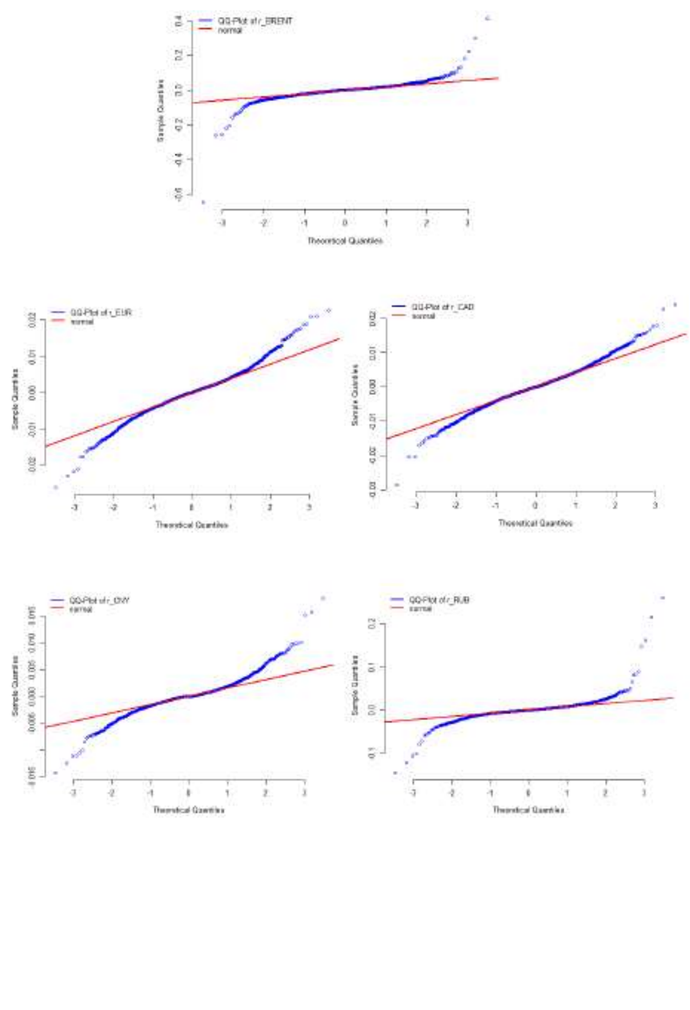

Fig. (2). QQ-Plot for daily returns.

Table 2. Jarque-Bera Test for the Returns.

	BRENT	EUR/USD	CAD/USD	RUB/USD	CNY/USD
T-Statistic	599777	426.94	315.16	429510	2811.9
P-value	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16

BRENT		EUR/USD		CAD/USD		RUB/USD	
BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC
-7728.5	-7750.8	-15687.4	-15715.1	-15771.6	-15799.1	-11046.8	-11069.2
-7728.2	-7756.2	-15681.8	-15709.9	-15771.3	-15799.3	-11041.75	-11069.81
-7725.4	-7758.9	-15679.6	-15713.3	-15764.6	-15798.3	-11040.64	-11074.32
-7729.1	-7757.1	-15683.8	-15711.9	-15772.4	-15800.5	-11042.9	-11070.96
-7744.1	-7777.6	-15676.2	-15709.9	-15772.7	-15806.4	-11068.41	-11102.09
-7736.9	-7776.1	-15672.2	-15711.5	-15765.2	-15804.5	-11076.87	-11116.16
-7725.6	-7759.1	-15679.5	-15713.2	-15765.2	-15798.9	-11039.89	-11073.57
-7736.8	-7776.1	-15672.1	-15711.4	-15765.2	-15804.5	-11077.54	-11116.83
-7729.3	-7774.1	-15670.1	-15709.8	-15758.3	-15803.2	-11070.27	-11115.18

CNY/USD		
	BIC	AIC
ARMA (1,1)	-17758.2	-17780.5
ARMA (1,2)	-17766	-17793.8
ARMA (1,3)	-17759.09	-17792.51
ARMA (2,1)	-17766.17	-17794.02
ARMA (2,2)	-17758.9	-17792.32
ARMA (2,3)	-17751.36	-17790.35
ARMA (3,1)	-17759.09	-17792.51
AR MA (3,2)	-17751.49	-17790.48
ARMA (3,3)	-17744	-17788.56

Table 3. The Selection of Model by Information Criteria: BRENT.

	GARCH	EGARCH	GJR-GARCH	APARCH
AIC	-4.6168	-4.6368	-4.7032	-4.6388
BIC	-4.6055	-4.6227	-4.6862	-4.6219
LogLikelihood (LL)	4560.816	4581.534	4648.079	4584.539

Table 4.The selection of model by Information criteria: EUR/USD

	GARCH	EGARCH	GJR-GARCH	APARCH
AIC	-7.9199	-7.9111	-7.9532	-7.9019
BIC	-7.9089	-7.8973	-7.9365	-7.8852
LogLikelihood (LL)	8018.984	8011.079	8054.603	8002.703

Table 5. The Selection of Model by Information Criteria: CAD/USD.

	GARCH	EGARCH	GJR-GARCH	APARCH
AIC	-7.8995	-7.9002	-7.9430	-7.8893
BIC	-7.8884	-7.8863	-7.9263	-7.8727
LogLikelihood (LL)	7998.255	8000.013	8044.296	7990.003

Table 6. The Selection of Model by Information Criteria: RUB/USD.

	GARCH	EGARCH	GJR-GARCH	APARCH
AIC	-6.4670	-6.4783	-6.6014	-6.4851
BIC	-6.4559	-6.4644	-6.5848	-6.4685
LogLikelihood (LL)	6548.613	6561.036	6686.613	6568.965

Table 7. The Selection of Model by Information Criteria: CNY/USD.

	GARCH	EGARCH	GJR-GARCH	APARCH
AIC	-9.1555	-9.2212	-9.5419	-9.2139
BIC	-9.1440	-9.2068	-9.5247	-9.1966
LogLikelihood (LL)	8880.233	8944.922	9256.914	8938.83

After estimating the different models, we conclude that the best models to be utilized are the ARMA(2,2) GJR-GARCH(1,1) for the BRENT, ARMA(1,1) GJR-GARCH(1,1) for EUR/USD, ARMA(2,2) GJR-GARCH(1,1) for CAD/USD, ARMA(3,2) GJR-GARCH(1,1) for RUB/USD and ARMA(2,1) GJR-GARCH(1,1) for CNY/USD.

Table 8. Estimation Results.

	BRENT	EUR/USD	CAD/USD	RUB/USD	CNY/USD
C	0.0021602	0.00016913	3.5869e-06	-0.0001359	-3.2613e-06
AR(1)	-0.81779	-0.66875	0.13143	-0.35309	-0.67787
AR(2)	-0.15018	-	0.78877	-0.11641	-0.050868
AR(3)	-	-		-0.042599	-
MA(1)	0.8369	0.70476	-0.1237	0.37648	0.61409
MA(2)	0.14937	-	-0.80326	0.10725	-
MA(3)	-	-	-	-	-
Constant $\boldsymbol{\omega}$	2.4888e-05	2e-07	2e-07	1.5584e-06	2e-07

GARH(1)($\boldsymbol{\beta}$)	0.85718	0.964894	0.94947	0.86484	0.85393
ARCH(1)($\boldsymbol{\alpha}$)	0.051864	0.046528	0.059458	0.17509	0.13657
Leverage	0.10378	-0.0050305	-0.032386	-0.09581	0.019011
Dof	5.3535	7.0072	7.1542	5.5121	2.8793

Table 9. Empirical Kendall's Tau τ Matrix for returns (Full Sample).

	BRENT	**EUR/USD**	**CAD/USD**	**RUB/USD**	**CNY/USD**
BRENT	1.00000000	-0.04937662	-0.24996598	-0.25604329	-0.09268464
EUR/USD	-0.04937662	1.00000000	0.29936901	0.13379245	0.25492254
CAD/USD	-0.24996598	0.29936901	1.00000000	0.2909988	0.2270470
RUB/USD	-0.25604329	0.13379245	0.2909988	1.00000000	0.1582451
CNY/USD	-0.09268464	0.25492254	0.2270470	0.1582451	1.00000000

The results show that all the test statistical of our variables are very higher than the critical value at the 5% level of significance, which leads to the rejection of the null hypothesis of normality inducing that all the distributions of the daily return series differ from the normal distribution.

4.3. Modeling Marginal Distributions

Modeling all returns and their volatilities consists of two steps. The first one involves the specification of the ARMA (1,1) model for mean returns. The second one is the specification of the GARCH (p, q) models for conditional volatility.

Conditional Mean

Our strategy for modeling the conditional mean is to search over alternative ARMA(p,q) models by varying p and q parameters from 1 to 3, and identifying the optimum model using the BIC and AIC.

The model, which has the lowest BIC and AIC, will be chosen. For the daily returns of the Brent, the best model is ARMA(2,2), for EUR/USD, the best model is ARMA(1,1), for CAD/USD, the best model is ARMA(2,2), for RUB/USD, the best model is ARMA(3,2), for CNY/USD, the best model is ARMA(2,1).

Conditional Variance

We used to explore the dynamics of volatility in presence of asymmetry effects for all our variables. We consider asymmetric GARCH models: GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1) and APARCH(1,1).

Table 8 shows that the parameter α ranges between 0.05 and 0.1,it is low and significant

The GARCH persistence parameter β is higher than 0.85 for all series, which means that it is significant and confirms the higher volatility. In addition, ARCH(1) coefficients are low and significant. The high α which is often associated with a low β produces GARCH volatilizes with a higher volatility of volatility. As a result, all series are described by significant GARCH effects.

4.4. Multivariate Analysis

The hypotheses that allow us to determine whether Brent can serve as a hedge and/or a safe haven against currencies are:

Hypothesis 1: $\tau_{Brent\,/i} \leq 0$: Brent is a hedge

Hypothesis 2: $\lambda_u = 0$ and/or $\lambda_L = 0$ Brent is a safe haven

Where $\tau_{Brent\,/i}$ is the Kendall tau between the Brent and other currencies (i)

λ_u and λ_L are the upper and lower tail dependence for the joint distribution of Brent and one of the other currencies.

The vine copula is a very useful model for dependency structures.

Build a Vine model using time-varying pair-copulas in the first tree, and condition only on the parameters of the time-varying pair-copulas in subsequent trees. The temporal properties of the dependency structure can be integrated into the pair copula parameters, allowing each conditional pair copula to vary over time when they belong to a family of parameter copulas.

First, we start with the Kendall's tau matrix as a selection criterion for trees; is a form of measure of concordance which measures dependence based on ranks. The sum of the absolute value of all Kendall's tau gives us the first root node variable for C-Vine copula, we choose the pair that captures the highest dependency in the first tree. It is summarized in the following table with scatter plot matrix:

Table **9** shows that the CAD/USD exchange rate has the greatest dependence in all cases.

Consequently, CAD/USD is the first root node (1), RUB/USD (2), EUR/USD (3), CNY/USD (4) and BRENT (5).

The table **9** and the fig. (**3**) show that the BRENT has a strong association with CAD/USD and RUB/USD exchange rates ($|\boldsymbol{\tau_{CAD}}|$=0.25 and$|\boldsymbol{\tau_{RUB}}|$=0.26). Whereas, the BRENT has a weak association with EUR/USD and CNY/USD. We also notice that there is an interrelation between the ex-

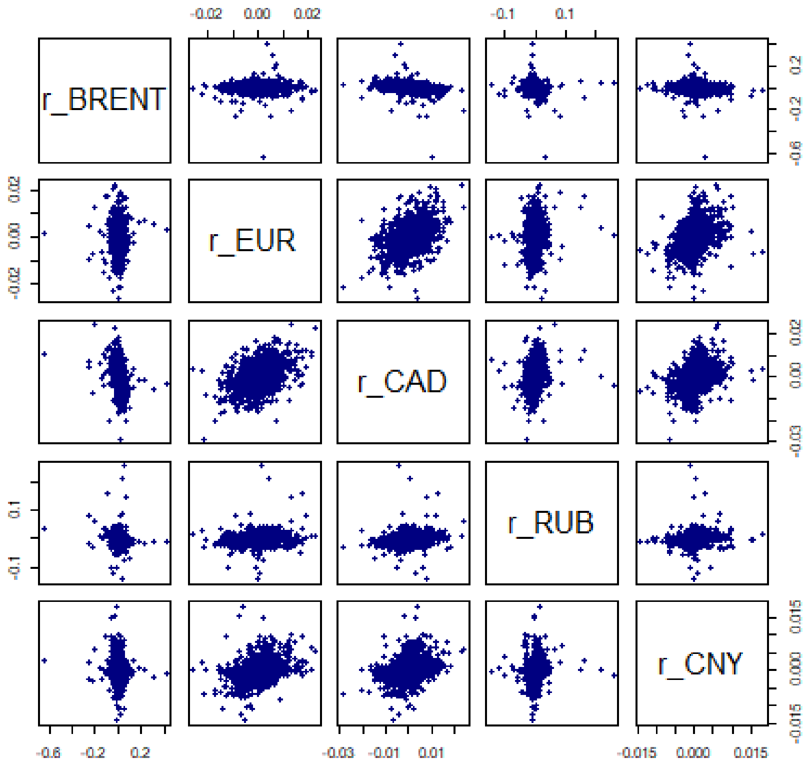

Fig. (3). Scatter plot matrix of the returns.

Table 11. Results of Estimated Parameters for the C-vine Copula (Full Sample).

Tree	**Edge**	**Family**	**Parameter1** θ_1	**Parameter2** θ_2	**Tau** τ	**Lower Tail dependence** λ_u	**Upper Tail dependence** λ_L
1	5,4	Rotated Clayton 270°	-0.18	0.00	-0.08	-	-
	5,3	t-Student	-0.07	9.49	-0.04	0.01	0.01
	5,1	t-Student	-0.39	13.55	-0.25	0.00	0.00
	5,2	t-Student	-0.39	3.59	-0.26	0.03	0.03
2	2,4;5	Frank	1.22	0.00	0.13	-	-
	2,3;5	t-Student	0.20	4.89	0.13	0.10	0.10
	2,1;5	t-Student	0.34	7.64	0.22	0.07	0.07
3	1,4;2,5	Gaussian	0.26	0.00	0.17	-	-
	1,3;2,5	t-Student	0.43	8.16	0.28	0.09	0.09
4	3,4;1,2,5	Frank	1.69	0.00	0.18	-	-

change rates, we note CAD/USD and EUR/USD have a strong relationship ($|\tau_{CAD\ EUR}|$=0.30), as well as CAD/USD and RUB/USD ($|\tau_{CAD\ RUB}|$=0.29). We conclude that there is an association between the BRENT and oil-exporting countries more than importing countries.

C-Vine Estimation

According to Czado, et al., 2012, first, we perform sequential maximum likelihood estimation to obtain initial values for the C-Vine copulas. Second, using the initial values from the first step, we apply maximum likelihood estimation (MLE) to estimate the final parameters of the paired copula.

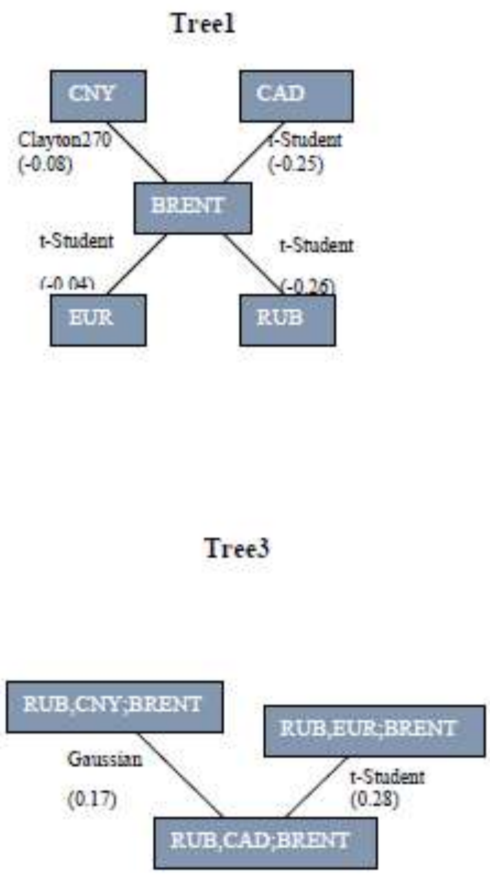

Fig. (4). Plot of C-Vine copula trees (Full Sample).

Table10 show parameter estimates for C-Vine with their trees.

The results of C-Vine show that most of the pairs give us t-student copula.

In the first C-Vine tree, we notice there is a weak negative dependence between Brent Crude Oil and EUR/USD and CNY/USD therefore, for the pairs Brent-CAD/USD and Brent-RUB/USD, the dependence is negative and somewhat greater than that of oil-importing countries. This indicates that Brent is a weak hedge and safe haven asset against currencies of oil importing countries, whereas it presents a strong hedge and a safe haven asset against currencies of oil exporting countries. Tree1 exhibits also a symmetric tail dependence given by student-t copula, exceptionally for the Rotated Clayton 270 degrees copula.

In the second tree, a positive dependence between all pairs with symmetric tail dependence given by t-student copula, exceptionally for the Frank copula between RUB/USD and CNY/USD conditional on Brent.

In tree 3, there is a positive dependence between EUR/USD and CAD/USD conditional on Brent and RUB/USD with a symmetric tail dependence given by student-t copula. In addition, we find a positive dependence between CNY/USD and CAD/USD, conditional on Brent and RUB/USD given by the Gaussian copula.

In tree 4, we notice that the dependence between EUR/USD and CNY/USD conditional on Brent, CAD/USD and RUB/USD is positive given by Frank copula.

Crisis Period Analysis

In this part, we will focus on two periods of crisis: The Covid-19 pandemic (from December 2019 to April 2021) and the invasion of Russia-Ukraine (from February 2022 to October 2022). We will redo the same approach of full sample for the two periods of crises.

The Covid-19 Pandemic:

The new order of nodes determined by the empirical kendall's tau matrix[1] is:

- Order(1) :CAD/USD
- Order(2) :RUB/USD
- Order(3) :CNY/USD
- Order(4) :BRENT/USD
- Order(5) :EUR/USD

Table **12** show parameter estimates for C-Vine.

[1] Empirical Kendall's Tau Matrix for returns during the covid-19 pandemic is in Appendix Table **6**

Table 12. Results of Estimated Parameters for the C-vine Copula (Period Covid-19).

Tree	Edge	Family	Parameter1 θ_1	Parameter2 θ_2	Tau τ	Lower Tail dependence λ_u	Upper Tail dependence λ_L
1	4,1	Rotated Gumbel 90°	-1.28	0.00	-0.22	-	-
	4,5	t-Student	-0.06	3.79	-0.04	0.07	0.07
	4,3	Rotated Joe 90°	-1.17	0.00	-0.09	-	-
	4,2	t-Student	-0.39	3.48	-0.25	0.03	0.03
2	2,1;4	t-Student	0.46	4.46	0.30	0.21	0.21
	2,5;4	t-Student	0.24	5.29	0.15	0.09	0.09
	2,3;4	Rotated Gumbel 180°	1.26	0.00	0.21	-	0.27
3	3,1;2,4	Frank	1.73	0.00	0.19	-	-
	3,5;2,4	Gaussian	0.34	0.00	0.22	-	-
4	5,1;3,2,4	Gumbel	1.33	0.00	0.25	0.32	-

Table 1. Results of Estimated Parameters for the C-vine Copula (Period Russia's Invasion of Ukraine).

Tree	Edge	Family	Parameter1 θ_1	Parameter2 θ_2	Tau τ	Lower Tail Dependence λu	Upper Tail Dependence λL
1	4,5	Gaussian	0.05	0.00	0.03	-	-
	4,3	Rotated Clayton 90°	-0.24	0.00	-0.11	-	-
	4,1	Frank	2.93	0.00	0.30	-	-
	4,2	Gaussian	0.49	0.00	0.33	-	-
2	2,5;4	Joe	1.20	0.00	0.10	0.21	-
	2,3;4	t-Student	-0.02	5.06	-0.01	0.05	0.05
	2,1;4	Gaussian	0.55	0.00	0.37	-	-
3	1,5;2,4	Rotated Gumbel 270°	-1.08	0.00	-0.08	-	-
	1,3;2,4	Frank	-2.18	0.00	-0.23	-	-
4	3,5;1,2,4	t-Student	0.07	3.71	0.04	0.10	0.10

The results of C-Vine show that most of the pairs give us t-Student copula.

From these tables, we note that even during the Covid-19 crisis, the dependence between Brent Crude Oil and all currencies remains weak and negative. We also note that Brent is more correlated with the exchange rates of oil-exporting countries (with Russian ruble -0.25 and -0.26 with Canadian dollar) than those of importing countries (-0.04 and -0.09 with EUR and the Chinese Yuan respectively).

In addition, we see that the dependence between RUB and CNY conditional on Brent remains positive but has increased during the Covid-19 crisis (from 0.13 to 0.21) with a lower tail dependence given by Rotated Gumbel 180 degrees copula, the same case for the dependence between RUB and EUR conditional on Brent and that between RUB and CAD conditional on Brent (from 0.13 to 0.15 and from 0.22 to 0.30 respectively). Hence, we note that the dependence between Brent and all currencies has increased during the Covid-19 crisis period.

Invasion of Russia-Ukraine

The new order of nodes determined by the empirical Kendall's tau matrixis

- Order(1) :CAD/US/USD
- Order(2) :EUR/USD
- Order(3) :BRENT/USD
- Order(4) :CNY/USD
- Order(5) :RUB/USD[2]

Table **13** show parameter estimates for C-Vine.

The table **13** show that the dependence between Brent Crude Oil and all currencies remains negative and has increased during the Russian invasion of Ukraine, exceptionally for the correlation between Brent and the Russian ruble which has

[2] Empirical Kendall's Tau Matrix for returns during the Russia's invasion of Ukraine is in Appendix Table **7**

become positive and very weak (from -0.26 to 0.06). We also note that the exchange rates have become more correlated during the crisis, for example the dependence between EUR and Canadian dollar increased from 0.30 to 0.44.

CONCLUSION AND DISCUSSION

From a global trade finance perspective, the oil-currency relationship is critical, as any significant movement in the price of crude oil or in the value of the U.S. dollar results in a joint movement in the major currency pairs widely used by oil exporters and oil importers. After the healthy and political, understanding the link between the foreign exchange and crude oil markets has important implications for traders and regulators. Therefore, investors and other market participants have been considering different risk management practices to reduce and diversify the risks in the two markets.

In this study, we required to examine the dynamic between Brent Crude Oil and currencies. In addition, the question arises whether crude oil prices can act as a hedge, safe haven and diversification factor for traditional currencies. For this reason, we adopted copula approach, which gives a better understanding of the dependence between the different financial instruments.

Results show that the Brent Crude Oil is negatively correlated with all currencies during the full sample period. More specifically, we find that Brent is more negatively correlated with Canadian dollar and Russian ruble than EUR and Chinese Yuan, which means that Brent is more negatively correlated with the exchange rates of the oil exporting countries (-0.25 with CAD and -0.26 for RUB) than those of oil importing countries. Hence, during the full sample period Brent has a strong hedge and safe haven against currency movements especially in the case of exporting countries (Canada and Russia).

During the Covid-19, the co-movements in major of the variables were higher than the full sample period and the dependence between Brent Crude Oil and all currencies remains negative, which confirms that Brent has a strong hedge and strong safe haven against currency movements.

For the Russia's invasion of Ukraine, we also find the combined movement of all variables is higher than the entire sample period, and the correlation between Brent crude oil and all currencies remains negative, exceptionally the dependence between Brent and Russian ruble remains positive and weak because of the crisis. As a result, we find that increases (decreases) in Brent Oil Prices in countries that are net oil exporters and importers are associated with the depreciation (appreciation) of that country's currency against the dollar in the full sample and during crisis, which means that a rise (decrease) in the oil price is related to a dollar appreciation (depreciation).

Finally, our results help traders identify appreciation and depreciation of currency in the foreign exchange market oil prices due to crisis and take advantage of the best trading opportunities. Our findings lead that Brent Crude Oil is always a strong hedge and safe haven against currency movements during volatile periods.

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