The Keynes Discontinuity Hypothesis, that J M Keynes Rejected his own Logical Theory of Probability and Accepted F P Ramsey's Subjective Theory, is based on the Myths Created by R B Braithwaite

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Abstract: Keynes realized that Boole had started the project of creating a logical foundation for the mathematical theory of probability. Boole made great progress in doing so in his 1854 The Laws of Thought. Unfortunately, he died at the early age of 49, having left his theory uncompleted.

Hailperin (1986) has pointed out that Boole's project was an unfinished one. The goal of Keynes's research program was to complete Boole's project. Keynes added further results in his research program by providing a much more detailed, propositional, relational logic to support probability theory, as well as introducing a new ,logical relation, which he designated as V, the evidential weight of the Argument. Keynes also provided an introductory, first order (predicate) logic supporting both the Principle of Indifference and Statistics.

The only economists, who realized what Keynes's goal was, was I. Hishiyama (1969, 2010). Unfortunately, Hishiyama did not have the skill set to pursue the question of how was it that Keynes was able to proceed and extend Boole's system in the manner that he did.

Starting in the late 1970's,academicians,especially philosophers and economists, who were ignorant of Hishiyama's point that there was a very dangerous, blind spot in the existing past and present assessments and interpretations of Keynes's A Treatise on Probability and General Theory, due to the failure of academicians to read Keynes's A Treatise on Probability in its entirety, started to consider what the connection was, if any, between the A Treatise on Probability and the General Theory and what was the nature of Keynes's accomplishments in the A Treatise on Probability. Unfortunately, the failure to absorb Hishiyama's point led the vast majority of academicians working on Keynes's 1921 book to ignore Keynes's Boolean framework and accept Ramsey's false claims. Current Keynes scholarship is infected by the belief that Ramsey had shown basic and fundamental flaws in the foundations of Keynes's relational, propositionallogic.

1. INTRODUCTION

All work done on Keynes's *A Treatise on Probability* by academicians, especially economists and philosophers, with a very few exceptions [for instance Edgeworth (1922), Russell (1922), Broad (1922), Hailperin (1986, 1965), Arthmar and Brady (2016, 2017) and Brady(2004 a,b)] in the 20th and 21st centuries, is fatally flawed by the complete ignorance of the basic and fundamental foundations for Keynes' s work. The foundations for Keynes's *A Treatise on Probability* (1921) are to be found in Boole's *The Laws of Thought* (1854; LT).

The total and complete failure to address this lacuna (or blind spot) was first recognized by I. Hishiyama in 1969, although Hishiyama was not able to specifically identify who the source was for Keynes new logic of decision making. Hishiyama referred to this as a blind spot .This blind spot has never been dealt with correctly by any economist or philosopher. The reason for this failure is the influence of the nefarious and malevolent claims made by F P Ramsey about Keynes's A Treatise on Probability in the period between January, 1922 and November, 1926.

The Ramsey influence was magnified many times over by the very questionable decision of an economist named Donald Moggridge, editor of Keynes's Collected Writings (CWJMK), to place an editorial foreword at the front of the 1973 CWJMK edition of Keynes's A Treatise on Probability, Volume8, written by a rabid Ramsey advocate and acolyte named R.B. Braithwaite. Braithwaite's editorial fore-

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word is simply a rewritten and toned down version that contains a number of Ramsey's errors and false assertions from his 1922 and 1926 reviews of Keynes's *A Treatise on Probability*. Anyone reading this editorial foreword will most likely be inclined to believe that the claims concocted by Ramsey are true .The only way to counteract this state of affairs would be to remove the Braithwaite foreword and replace it with the assessments of either Edgeworth (1922), Russell (1922), or Hailperin (1986).

Unfortunately, this interest in Keynes' work was pursued by academicians who did not have the training, expertise and skillset in formal mathematical logic, mathematics, probability and statistics that would be required in order to allow them to successfully read ,understand and comprehend the technical, detailed mathematical analysis provided by Boole in *The Laws of Thought* and by Keynes in the *A Treatise on Probability*, which was based on *The Laws of Thought* .Given that Keynes's foundation for the *A Treatise on Probability* was built upon his version of Boole's relational, propositional logic, Boole's objective ,logical probability relation and Boole's interval valued approach to probability, a necessary requirement for success would require that the Boole-Keynes connection be recognized at the very start of any research project that intended to engage in a careful and complete study of the A Treatise on Probability. This has never occurred.

A study of the work done over the last 30-45 years of R. O'Donnell, A. Carabelli, B. Davis ,B. Bateman and J. Runde, who are the main advocates of the Keynes Discontinuity Hypothesis, which is that Keynes changed his mind about probability ,rejected his own theory and adopted Ramsey's theory, reveals that none of these writers had the slightest inkling/idea/understanding of the Boolean logical foundations supporting Keynes's work in the A Treatise on Probability or how Keynes's General Theory was built on the A Treatise on Probability. This extreme and extraordinary ignorance shows up repeatedly whenever the work of F P Ramsey is considered as somehow being relevant to the question of what were Keynes's accomplishments in the A Treatise on Probability and General Theory. Ramsey's incredibly flawed work must be judged as being totally irrelevant to any consideration of what Keynes's unique and original accomplishments were in his A Treatise on Probability and General Theory

Similarly, anyone reading the editorial foreword written by R. B Braithwaite ,has, very likely , already severely intellectually hamstrung themselves because everything talked about by Braithwaite in the foreword is completely irrelevant to any assessment of Keynes's contributions. Braithwaite never mentioned the role played by G Boole in Keynes's research project in the *A Treatise on Probability* during his lifetime just as Ramsey never mentioned Boole's name.

Given the complete lack of any understanding whatsoever of what the nature of Boole's contributions to Keynes's magnum opus were, the combined work of all five of the academicians listed above need to be completely discarded as being totally irrelevant as far as making any assessment of or consideration of the value of Keynes's contributions to probability, statistics and decision theory in the *A Treatise on Probability* and *General Theory*.

Boole is never mentioned in any of the work done by R. O'Donnell, A. Carabelli, B. Davis, B. Bateman and J. Runde, who are the main propagators of the Keynes Discontinuity Hypothesis. It is not possible to understand and grasp Keynes's logical theory of probability unless a reader has a firm grasp of what and how Boole contributed to Keynes's research program in probability and statistics.

The Keynes Discontinuity Hypothesis immediately collapses once it is realized that Ramsey's critiques of Keynes's logical theory of the *A Treatise on Probability* make no sense. See my discussion of Bateman below for the details of Ramsey's many errors and mistakes.

This paper will be organized in the following manner. Sections Two and Three will present Boole's and Keynes's approaches .This will entail chapters I, XI, XII and XVI-XXI of Boole's work and chapters I,II and X -XVII of Keynes's A Treatise on Probability. We will then show in sections Four through Eight that NONE of this material is mentioned, understood, graspedor dealt with correctly by any of the main advocates of the Keynes Discontinuity Hypothesis (KDH) that I have selected to examine -O'Donnell (1989, 2021a,b), Carabelli (1988, 2003), Davis (1994, 2021), Runde (1994) or Bateman (1987, 1989, 1990, 1992, 2016, 2021)- over the last 40 plus years. Of course, it needs to be said that practically all economists or philosophers, who have written on Keynes's logical theory, would also be shown to be severely deficient intellectually as regards their grasp of Keynes's theory.

Section Nine concludes that academicians are still stuck at the 1969 level of understanding, which was identified by Hishiyama as a blind spot. Hishiyama correctly and shrewdly identified what the problem was -a failure to read Keynes's A Treatise on Probability. I have come to exactly the same conclusion as Hishiyama did in 1969 in 2022.Keynes's Treatise has simply not been read in its entirety .Generally academicians read some parts taken from Parts I and III of Keynes's A Treatise on Probability that does not require any of the extensive logical, mathematical and statistical training which is necessary to understand Keynes's Boolean approach.

2. BOOLE'S ORIGINAL RELATIONAL, PROPOSITIONAL LOGIC OF THE LT

Boole explains his approach in a straightforward and direct manner:

"Logic is conversant with two kinds of relations,–relations among things, and relations among facts. But as facts are expressed by propositions, the latter species of relation may, at least for the purposes of Logic, be resolved into a relation among propositions. The assertion that the fact or event A is an invariable consequent of the fact or event B may, to this extent at least, be regarded as equivalent to the assertion, that the truth of the proposition affirming the occurrence of the event B always implies the truth of the proposition affirming the occurrence of the event A. Instead, then, of saying that Logic is conversant with relations among things and relations among facts, we are permitted to say that it is concerned with relations among things and relations among propositions.

Of the former kind of relations we have an example in the proposition–"All men are mortal;" of the latter kind in the proposition–"If the sun is totally eclipsed, the stars will become visible." The one expresses a relation between "men" and "mortal beings," the other between the elementary propositions–"The sun is totally eclipsed;" "The stars will become visible." Among such relations I suppose to be included those which affirm or deny existence with respect to things, and those which affirm or deny truth with respect to propositions. Now let those things or those propositions among which relation is expressed be termed the elements of the propositions by which such relation is expressed. Proceeding from this definition, we may then say that the premises of any logical argument express given relations among certain elements, and that the conclusion must express an implied relation among those elements, or among a part of them, i.e. a relation implied by or inferentially involved in the premises."(Boole, 1854, p.7).

It should be obvious now that the claims made by Ramsey in 1922 and 1926 ,that such logical relations between the elementary propositions of an argument do not exist ,is pure nonsense that Boole would have punished with a withering attack on Ramsey's reviews ,which was Boole's response to every published attack on LT.

Boole's intention then, to apply his relational, propositional logic to probability, first appears on p.12 of the LT:

"The general doctrine and method of Logic above explained form also the basis of a theory and corresponding method of Probabilities. Accordingly, the development of such a theory and method, upon the above principles, will constitute a distinct object of the present treatise. "(Boole, 1854, p.12).

On page 17, we are given a summary of this approach:

"As the method is independent of the number and the nature of the data, it continues to be applicable when the latter are insufficient to render determinate the value sought. When such is the case, the final expression of the solution will contain terms with arbitrary constant coefficients. To such terms there will correspond terms in the final logical equation (I. 15), the interpretation of which will inform us what new data are requisite in order to determine the values of those constants, and thus render the numerical solution complete.

If such data are not to be obtained, we can still, by giving to the constant their limiting values 0 and 1, determine the limits within which the probability sought must lie independently of all further experience. When the event whose probability is sought is quite independent of those whose probabilities are given, the limits thus obtained for its value will be 0 and 1, as it is evident that they ought to be, and the interpretation of the constants will only lead to a restatement of the original problem.

2ndly. The expression of the final solution will in all cases involve a particular element of quantity, determinable by the solution of an algebraic equation. Now when that equation is of an elevated degree, a difficulty may seem to arise as to the selection of the proper root…the numerical limits, within which the probability sought must have been confined, if, instead of being determined by theory, it had been deduced directly by observation from the same system of phaenomena (sic) from which the data were derived. It is clear that

these limits will be actual limits of the probability sought. Now, on supposing the data subject to the conditions above assigned to them, it appears in every instance which I have examined that there exists one root, and only one root, of the final algebraic equation which is subject to the required limitations."(Boole, 1854, p.18).

It is at this point in the development of his new approach that Boole first introduced the concept of imprecise probability.

We now move to chapter XVI, which is Boole's introduction of how propositions about events will replace events in an analysis of probability, which Boole based on chapters I-XV of LT. Thus, instead of numbers, Boole will use as basic building blocks propositions:

"Before we proceed to estimate to what extent known methods may be applied to the solution of problems such as the above, it will be advantageous to notice, that there is another form under which all questions in the theory of probabilities may be viewed; and this form consists in substituting for events the propositions which assert that those events have occurred, or will occur; and viewing the element of numerical probability as having reference to the truth of those propositions, not to the occurrence of the events concerning which they make assertion. Thus, instead of considering the numerical fraction p as expressing the probability of the occurrence of an event E, let it be viewed as representing the probability of the truth of the proposition X, which asserts that the event E will occur. Similarly, instead of any probability, q, being considered as referring to some compound event, such as the concurrence of the events E and F, let it represent the probability of the truth of the proposition which asserts that E and F will jointly occur; and in like manner, let the transformation be made from disjunctive and hypothetical combinations of events to disjunctive and conditional propositions. Though the new application thus assigned to probability is a necessary concomitant of the old one, its adoption will be attended with a practical advantage drawn from the circumstance that we have already discussed the theory of propositions, have defined their principal varieties, and established methods for determining, in every case, the amount and character of their mutual dependence."(Boole, 1854, p.247)

The reader is advised to go to page 5 of Keynes's A Treatise on Probability in chapter I where he will find an identical discussion made by Keynes, where a footnote on page 5 is attached explicitly links Keynes's propositional logic to that of George Boole.

3. KEYNES'S "NEW" RELATIONAL, PROPOSITIONAL LOGIC OF THE TP

It is on page 5 of chapter I of the TP that Keynes reveals what this new approach entails:

"With the term "event," which has taken hitherto so important a place in the phraseology of the subject, I shall dispense altogether.† Writers on Probability have generally dealt with what they term the "happening" of "events." In the problems which they first studied this did not involve much departure from common usage. But these expressions are now used in a way which is vague and ambiguous; and it will be more than a verbal improvement to discuss the truth

and the probability of propositions instead of the occurrence and the probability of events.‡"(Keynes,1921,p.5).

The footnote that Keynes appends explains exactly what work Keynes is going to base his work on:

"‡The first writer I know of to notice this was Ancillon in Doutes sur les bases du calcul des probabilités (1794): "Dire qu'un fait passé, présentouà venirest probable, c'est dire qu'une proposition est probable." The point was emphasized by Boole, Laws of Thought, pp. 7 and 167. See also Czuber, Wahrscheinlichkeitsrechnung, vol. i. p. 5, and Stumpf, Über den Begriff der mathematis chen Wahrscheinlichkeit." (Keynes , 1921, p.5).

One can easily eliminate Ancillon and Stump as being innovators in this field. One is then left only with George Boole .It has been universally accepted for over a century that it was George Boole who created the original relational ,propositional logic .Keynes did not create a new logic .He applied the new and original logic of Boole .Keynes then greatly extended the applicability of Boole's original ,logical system in the A Treatise on Probability(TP,1921) and later in the General Theory(GT,1936).

Keynes is correct in his footnote on p.5 that on p.7 of the LT Boole first describes what a relational ,propositional logic will entail and is correct that on p.167 Boole states ,again ,that this approach deals with propositions about events and not the events themselves ,as well as differentiating between primary and secondary propositions and the truth and falsity of propositions .However ,it is apparent that no reader of the TP grasped what this meant as pointed out by Hishiyama in 1969.

Keynes gives a summary of what this means as far as Keynes's approach is concerned:

"9. This chapter has served briefly to indicate, though not to define, the subject matter of the book. Its object has been to emphasize the existence of a logical relation between two sets of propositions in cases where it is not possible to argue demonstratively from one to the other. This is a contention of a most fundamental character. It is not entirely novel, but has seldom received due emphasis, is often overlooked, and sometimes denied. The view, that probability arises out of the existence of a specific relation between premiss and conclusion, depends for its acceptance upon a reflective judgment on the true character of the concept. It will be our object to discuss, under the title of Probability, the principal properties of this relation. First, however, we must digress in order to consider briefly what we mean by knowledge, rational belief, and argument."(Keynes,p.9).

Unfortunately ,Keynes's"… contention of a most fundamental character." has eluded philosophers and economists ,as pointed out by Hishiyama ,because they do not know what Keynes's relational ,propositional logic is based on -an extension and development of Boole's original relational, propositional logic of 1854 in LT.

As an aside, we can now be in a position to also dispose of Ramsey's strange and bizarre claim that this type of formal logic can involve only two propositions at a time .Where this wild and unsupportable claim of Ramsey's came from can't be identified .All that can be said is that it does not ap-

pearany where in Keynes's TP .Nowhere in Keynes's TP will any researcher find that Keynes had restricted the application of his (Boolean) relational ,propositional logic to two propositions only:

"Let our premises consist of any set of propositions h, and our conclusion consist of any set of propositions a, then, if a knowledge of h justifies a rational belief in a of degree α, we say that there is a probability-relation of degree α between a and h.∗" (Keynes,1921,p. 4; see p. 5 for the requirement that there must be a connection between the h and a propositions for any logical relation to connect the sets of propositions and pp. 52-56 that the propositions must be relevant to each other and not irrelevant.) and "Between two sets of propositions, therefore, there exists a relation, in virtue of which, if we know the first, we can attach to the latter some degree of rational belief. This relation is the subject-matter of the logic of probability.

A great deal of confusion and error has arisen out of a failure to take due account of this relational aspect of probability."(Keynes, 1921, p.6).

Thus, it is easy to provide a complete refutation of both of Ramsey's queer reviews of 1922 and 1926 just with a knowledge of the first 6 pages of chapter I.

I will repeat Keynes's page 5 assessment because of its extreme importance. The failure to grasp this paragraph explains much of the quagmire the economics and philosophy professions now find themselves in:

"With the term "event," which has taken hitherto so important a place in the phraseology of the subject, I shall dispense altogether.† Writers on Probability have generally dealt with what they term the "happening" of "events." In the problems which they first studied this did not involve much departure from common usage. But these expressions are now used in a way which is vague and ambiguous; and it will be more than a verbal improvement to discuss the truth and the probability of propositions instead of the occurrence and the probability of events. ‡"(Keynes,1921,p.5).

I believe it is self evident that Boole and Keynes are using the same approach, a relational, propositional logic which Frank Ramsey was totally ignorant about. However, it is the case that even a powerful mathematician and logician like Borel (1924), admitted that he was skipping Part II of the TP in his review.

4. CARABELLIAND THE KEYNES DISCONTINUITY HYPOTHESIS

Consider the following set of statements made by Carabelli: 3 Key Doctrines Concerning Probability.

3.1. Direct and Indirect Knowledge

Abstract

3.1.1. Keynes's view of probability, whose basic aspects were considered in the previous chapter, was centred on some general key doctrines. As I have already noted, these doctrines were not always explicit and expressed in univocal and coherent form. Hence the necessity not only of a close reading of Keynes's text, but also of a sort of systematic re-

construction of Keynes's approach to key epistemological topics, together with an attempt to clarify his position within its historical intellectual context. Such a task, which will be attempted in the present section, will enable one, for instance, to grasp the fact overlooked in a superficial reading of the Treatise, that Keynes (as we will see in Chapter 8) did not usually adopt the term 'logical' in the sense of formal logic, but in the sense of ordinary language logic, that is, in a sense which was actually antithetical to it. This explains the above-mentioned uncritical ranking of Keynes within the so-called logicist approach to probability." (Carabelli, 1988, p.23).

"...the logicist interpretation of Keynes's theory appears to be based on a hasty reading of Keynes's text. In various passages Keynes did indeed speak of the "logical "character of his notion of probability. But this fact does not mean that...it was a logic of the formal type. Infact, it was an ordinary discourse logic."(Carabelli,1988, p. 145).

"In his famous critique of A Treatise on Probability, Frank Ramsey threw doubt on the existence of the logical probability relations on which Keynes's theory of probability is founded. Keynes admitted that there was something to Ramsey's complaint, and allowed that the basis of our degree of beliefs rather than corresponding to objective logical relations, may be part of our human outfit….But Keynes clearly continued to adhere to the contrast drawn in A treatise on probability between actual beliefs on the one hand and reasonable[author's note -Keynes used the word rational] beliefs on the other."(Carabelli, 2003, p.223).

Contrary to Carabelli, Keynes had always been a formalist. Keynes had always been a logicist. Keynes had ALWAYS made it clear that Ramsey's approach could NEVER deal with rational belief. Ramsey NEVER threw doubt on the existence of logical probability relations as far as Keynes was concerned. Keynes NEVER admitted that there was something to Ramsey's complaint. Keynes NEVER allowed in his system of logical probability dealing with rational beliefs "that the basis of our degree of beliefs, rather than corresponding to objective logical relations, may be part of our human outfit." This only held in Ramsey's subjective theory system, not Keynes's .

Carabelli's severe confusions can be explained by her absorption of the Braithwaite editorial foreword that she read repeatedly at Cambridge University, England during the time she was doing her dissertation on Keynes under Donald Moggridge, who was responsible for the publication of Braithwaite's nonsense at the beginning of the TP.

Just how did Ramsey "throw doubt" on the existence of the logical probability relations on which Keynes's theory of probability is founded, given that these logical probability relations are IDENTICAL to those of Boole ? Carabelli has never provided an answer to this question so far.

5. O'DONNELL AND THE KEYNES DISCONTINUITY HYPOTHESIS

O'Donnell completely overlooks the fact that Keynes's logical framework comes directly from Boole. O'Donnell writes about "Keynes's logical framework" in complete ignorance of the fact that Keynes's framework is built directly on Boole. Consider the following examples of O'Donnell's delusions and illusions about Keynes's authorship of a new logic of belief:

"Keynes's conciliation consisted in bringing the family of non -conclusive but rational arguments under the umbrella of a reformulated logic."(O'Donnell, 1989, p.30).

In what way, then, was logic to be generalized? Keynes's answer was to make probability the foundation of a new general theory of logic…."(O'Donnell,1989,p.30).

"Once Keynes's understanding of probability is grasped in this wider senses-as a general theory of logic, the purpose of which is a universal explanation of rational inference -…."(O'Donnell,1989,p.31)

"Novelty was something upon which Keynes often remarked, not in frequently in relation to his own work. In 1906 he wrote to his parents, 'My method is quite new (XV 2) a sentiment echoed both in his second dissertation(1908 p.ii)-In detail there is much that is old, but the general structure and the pervading conception …. are claimed as original'-and in the Preface to the TP(xxv-[note-the reader should note that there is no such page xxv in the Preface to the A Treatise on Probability)."(O'Donnell, 1989, p.31).

O'Donnell comes very close to identifying the source of Keynes's foundation, but apparently was oblivious to the clue:

"It is, however, novelty within a long tradition, the idea of logic as the science of reason and 'the laws of thought'… Keynes continued this broad tradition, his goal being to expand 'the laws of thought' to include probable as well as conclusive inference"(O'Donnell,1989,p.31).

O'Donnell apparently has never realized, in 1989 or at anytime in his life, that Boole's 1854 book is titled "The Laws of Thought".

However, this lacuna in O'Donnell's work should have been rectified when he came to read p.5 of the TP, especially footnote 2 where Keynes makes it very clear that Boole was the first to have emphasized that it was a proposition about an event rather than the event itself ,which would be fundamental in his analysis. O'Donnell's failure to recognize what the Boolean foundations of Keynes's approach calls into question the relevance of his entire dissertation.

We now move to O'Donnell's flawed understanding of Keynes's relational, propositional logic:

"Probability, for Keynes, is essentially about logical relations between sets of propositions Broadly stated, the doctrine applies to any pair of sets, but its particular relevance is to those pairs that constitute the premises and conclusions of arguments."(O'Donnell, 1989, p.34),

Of course, it does not apply to any pairs of sets. It only applies to sets of a and h propositions that satisfy Keynes's relevance-irrelevance logic of pp.52-56 of the TP and /or propositions that are "logically connected "(Keynes, TP, p.5),"afford grounds for "(Keynes, 1921, p.4) or "yields some grounds for believing it" (Keynes, 1921, p.5). This most egregious error is identical to Ramsey's error, an error that permeated all of Ramsey's writing on Keynes's logical theory of probability.

O'Donnell appears to have had second thoughts on p.40:

"It is not completely accurate to represent Keynes's probabilities as applying to *any* (O'Donnell's emphasis) two sets of propositions. Certain conditions, mentioned below laid on a and h…"(O'Donnell,1989,p.40). However, three of the supposed qualifications miss the point entirely. These are a) that h represents all of the knowledge in the mind of the rational individual; b) that h will typically vary between individuals; and c) that this is associated with truth in two possible ways.

O'Donnell's last qualification is d) premisses should not be 'self-contradictory or formally inconsistent with themselves.

This last qualification was, of course, also included by Ramsey in 1922 (Ramsey, 1922, p.3).

Nowhere does O'Donnell state that there must be a logical connection between the a and h propositions or that the a and h propositions must satisfy the relevance -irrelevance logic presented by Keynes on pp. 52-56 of the TP. Infact, O'Donnell has NO KNOWLEDGE of why Keynes's relevance -irrelevance logic, as used by B. Russell on p.120 in his review of Keynes's TP in the 1922 Mathematical Gazette, totally refutes BOTH of Ramsey's reviews completely. O'Donnell claims to have read Russell's reviews, but only cites from p. 119 and p.125.

There is no difference between O'Donnell and Ramsey over how they view Keynes's relational propositional logic. O'Donnell simply remains silent in his chapter 2, never mentioning that his discussion amounts to a complete and total capitulation to Ramsey's claims about Keynes's relational propositional logic, which comes from Boole.

It is in chapter 7, pp.139-147 of O'Donnell's 1989 book ,that one finds that O'Donnell simply does not grasp that Keynes's Boolean relational, propositional logic is constrained by the logic of relevance -irrelevance, initially presented by Keynes on pp.52-56 before being presented in advanced form in Part II of theTP in chapters 12 and 14.O'Donnell starts his position in the following manner in his discussion of Keynes's review of Ramsey in the October, 1931 issue of the *New Statesman and Nation:*

"Keynes's concession to Ramsey, whatever its nature, did not constitute a radical shift in the foundations of his thought. The review is not without significance, however, One can accept that Ramsey disturbed Keynes in some sense."(O'Donnell 1989, p.140).

What disturbed Keynes was Ramsey's incredible ignorance of his Boolean relational, propositional logic; given that Ramsey was a genius, Keynes was probably completely flabbergasted, as was Bertrand Russell, over Ramsey's erroneous critique of this logic.

O'Donnell presents an above average explanation of why Keynes did not capitulate to Ramsey, given his ignorance of the fundamental logical flaws in Ramsey's position, on pp.141-142.

However, he immediately destroys the value of this defense of Keynes on pp.143-147:

"But if the evidence shows that Keynes did not actually abandon the framework of his logical theory, why did he imply in 1931 that he had ?" (O'Donnell, 1989, p.143)

Nowhere in Keynes's review of Ramsey ,which is all about Ramsey's theory, which has nothing to do with Keynes's theory, which was built on interval valued probability and decision weights ,does Keynes imply that he was abandoning his theory. Keynes's theory was built on an impregnable foundation of Boolean logic and algebra .The development of Keynes's logical analysis in Part II of the TP leads inevitably to imprecise probability constructs, where non additivity explains uncertainty.

The real problem here is that O'Donnell has accepted Ramsey's claim about Keynes's theory being an ordinal probability theory. O'Donnell has absolutely no understanding of any kind of interval valued concept of probability and/or decision weight approach, which is the only way to deal with non additivity, which MUST OCCUR if the evidential weight of the evidence, V(a/h)= w ,has a w value of <1,where w is defined on the unit interval as is probability as defined by Keynes on p. 315 of the TP. It is true that, if you do not understand the Keynes-Boole connection concerning interval probability, then you are susceptible to Ramsey's false claims about Keynes's theory being an ordinal one.

O'Donnell continues to pursue his erroneous approach, which leads to the more erroneous conclusions as he writes the rest of his chapter 7:

"The second main element is the development of Keynes's feeling of vulnerability towards Ramsey. In 1922 it was hardly evident-he respected Ramsey as a damaging critic opposed to the logical account."(O'Donnell, 1989, p.144).

O'Donnell never comes right out and says how Ramsey's account of Keynes's Boolean relational propositional logic was damaging, in Keynes's view, given Russell 's 1922 refutation of Ramsey's attack on p.120 of the Mathematical Gazette in July of 1922 in a single tiny footnote ,which was based directly on pp.52-56 of the TP!

O'Donnell continues to pile one error after another that then leads to more errors:

"But as time passed …his feeling of vulnerability grew." (O'Donnell, 1989, p.144)

How is Ramsey's fundamentally flawed logical argument going to be able to make Keynes feel more vulnerable? O'Donnell gives no answer. Instead, he quotes a letter, written to Urban in 1926, that has nothing to do with Ramsey at all.

O'Donnell has now build his entire assessment of Keynes's reply to Ramsey on a large number of interconnected errors that make it is impossible for him to recover :

"Here his expressed vulnerability is towards the frequency theory, but Ramsey's shadow is still detectable …"(O'Donnell,1989,p.145).

I can't detect any such shadow, especially given Keynes's devastating critique of Tinbergen's frequentist approach to probability in 1938-1940, a critique that still holds as of 2022.

O'Donnell then proceeds to agree that Keynes capitulated to Ramsey:

"The consequence was that his surrender on paper was entirely misleading …What ,then, was the real pull of Ramsey's thought on Keynes ?...firstly, a collapse of Keynes's conviction that the approach of the TP enjoyed unchallenged supremacy; secondly, a partial acceptance of some of Ramsey's strictures about the precision of ideas; and thirdly ,encouragement towards an *internal* shift within Keynes's philosophical framework."(O'Donnell, 1989, p.145).

How O'Donnell is able to conclude that Keynes believed that the TP "enjoyed unchallenged supremacy" is completely unclear as every single frequentist ,who commented on it or reviewed it, rejected it totally with the exceptions of Crum, E. Wilson (1923) in a second ,disguised review in 1934 that came 11 years after his 1923 review ,and E. Borel (1924) in a second review in 1939 that came 15 years after his 1924 review.

The other points made by O'Donnell are irrelevant because he is fundamentally ignorant of the basic nature of how Keynes's logical theory of probability is operationalized, which is, like Boole, by interval valued probability and Keynes's original contribution, decision weights .

O'Donnell's conclusion, that "At Cambridge, Ramsey's subjective probability theory undoubtedly grew out of his inability to cognize Keynes's objective relations." (O'Donnell, 1989, p.147) is irrelevant. Ramsey's real problem was his failure to understand what Keynes's relational ,propositional logic involved -propositions that were logically connected, associated or related to each other and/or propositions that were relevant and not irrelevant, as every example given by Ramsey from 1922-1926 involves ,as demonstrated simply by Russell(1922),propositions which are NOT connected ,associated ,related and/or relevant.

Ramsey's horrible performance in his reviews of Keynes's TP in 1922 and 1926, as well as in all of his private papers and private correspondence with Keynes between 1922 and 1926,lead one to the conclusion that it is impossible to view Ramsey as the greatest philosopher -logician of the 20th century as implied by C Misak (2020a,b,2016).

6. RUNDE AND THE KEYNES DISCONTINUITY HYPOTHESIS

Runde's severe and repeated errors, all based on his acceptance of the Ramsey myth's spread by R. B. Braithwaite, was taught to him by way of the Braithwaite editorial foreword at the front of the 1973 CWJMK edition of the TP. This edition of the TP was used to teach students in undergraduate and graduate level philosophy and economics courses in the economics department at Cambridge University, England. It explains how it was that Runde came to accept the Ramsey myth.

Consider the following statements by Runde:

"…many would argue that anything worth preserving in this work [author's note –Runde is referring to Keynes's TP] must surely have been incorporated in the prevailing wisdom and that the balance must simply be wrong or at best irrelevant. And finally ,there is the matter of Frank Ramsey's

(1931) famous critique of the Treatise .It is widely held ,not only that Ramsey's arguments are decisive ,but that Keynes yielded to them(e.g. Mellor, 1983, p.10;1995).If the author of the logical approach was himself persuaded to give up on it, why attempt to resurrect it now ?"(Runde, 1994, p.97).

Of equal concern is Runde's reliance on the intellectually defective 1973 CWJMK version of the TP in volume 8.The edition used by Runde contains the Braithwaite editorial foreword, with its many intellectual land mines that lead a new reader of the TP to already conclude that Keynes was wrong and Ramsey was right before that reader has even begun to read the TP.

The result is that Runde simply accepts the false claims that Keynes rejected the foundation of the TP, carefully constructed by Keynes in chapters I, II and III, of his logical relation P (see page 119 of the TP,1921),

where

P(a/h)=α,where 0≤α≤1,and α is a degree of rational belief.

P is the objective, logical relation that connects the related relevant a and h propositions by some degree of similarity, which is what Keynes defined his objective logical relation to be.

Runde is very clear that, in response to Ramsey's reviews, Keynes supposedly agreed that there was no possible way that a decision maker could judge if there is any connection between the h and a propositions. Runde concludes that Keynes capitulated to Ramsey in his *New Stateman and Nation* article published in the Oct., 1931 issue:

"This is straightforward enough and is widely interpreted as a unilateral surrender on Keynes's part."(Runde, 1994, p.107).

The problem here is that Runde has no idea that Keynes is not talking about his theory of rational degrees of probable belief, based on imprecise, interval valued, non numerical, probability, but about Ramsey's precise, numerical valued theory of degrees of belief .Keynes correctly admitted only that Ramsey's precise theory is a better explanation for the purely mathematical laws of the probability calculus than the frequentist theory.

Runde's page 107 assessment represents a complete and total capitulation on the part of Runde to both Ramsey (and B. Bateman) about how Keynes capitulated to Ramsey. Runde clearly aligns himself with Bateman:

"Keynes clearly revokes this commitment in the passage quoted above, and adopts Ramsey's view that the basis of our degrees of belief is simply 'part of our human outfit, analogous to our perceptions and our memories rather than to formal logic'. Our rational beliefs, then ,are not founded on directly intuited RPI's "(Runde, 1994, p.108).

Of course, Keynes never argued that "degrees of belief "were based on PRI's. Keynes argued that rational degrees of belief are based on RPI's. It is Ramsey's subjective probabilities that are"… based on part of our human outfit, analogous to our perceptions and our memories rather than to formal logic'.

Runde has made quite an incredible and incomprehensible logical blunder here. Keynes's "concession "concerns Ramsey's degrees of belief, not Keynes's degrees of rational belief. Keynes's theory deals with rational degrees of belief, not Ramsey's degrees of belief.

It is logically impossible, then, for Keynes to state that our rational degrees of belief are not based on RPI's because Keynes would then have to repudiate the Boolean based relational, propositional logic that is universally accepted by logicians as the foundation of all of mathematical logic. Runde has totally confused Keynes's rational degrees of belief with Ramsey's degrees of belief.

Runde's 1994 paper does not contain any defense of Keynes's logical theory of probability or his A Treatise on Probability at all. It is just a different variation on a theme as presented by Braithwaite. One could view it as a sugar coated version of Braithwaite's 1973 editorial foreword .

The only correct part of Runde's article is his comments on Keynes's introductory comments on comparative probability which Keynes used to under pin his Principle of Indifference. Everything else in Runde's article, as is also the case in Braithwaite's foreword and Ramsey's 1922 and 1926 comments ,are just plain wrong.

Section 7.Bateman and the Keynes Discontinuity Hypothesis

B. Bateman's assessment of the current state of understanding about Keynes's A Treatise on Probability in 2021 ,one hundred years after Keynes published his magnum opus, is that "… we are still faced with unresolved, fundamental questions about his foray into the philosophy of probability. One of these unresolved questions concerns whether Keynes (1931) later changed his mind in response to intense criticism from Frank Ramsey (1922, 1931) and abandoned the logical theory of probability."(Bateman, 2021, p.619).

In fact, there are no unresolved questions or concerns concerning the origins, theoretical development and technical applications of Keynes's logical theory of probability in his A Treatise on Probability if a reader understands two important conclusions, which can only be obtained by a careful and prolonged reading of the A Treatise on Probability over a number of years.

First, such a reader will have understood that Keynes's logical theory of probability is built upon G. Boole's logical theory of probability .Boole was the first to technically develop the relational, propositional logic used, and improved upon, by Keynes in his book. Keynes's use of a relational, propositional logic follows directly, and builds upon, Boole's approach .It has nothing to do with Platonic, metaphysical entities that supposedly reside in some imaginary, immaterial world .

Second, such a reader will come to the conclusion that F P Ramsey's two reviews of Keynes's A Treatise on Probability, in 1922 in the January issue of Cambridge Magazine and in 1926 in his "Truth and Probability", (published in 1931; see Kyburg and Smokler (eds.), 1980), are completely worthless, as far as Ramsey's claims about Keynes's logical Theo-

ry of probability go. Ramsey's claims have nothing to do with anything that is actually contained in Keynes's book. Ramsey makes up definitions that are imaginary constructs that can easily be seen by a reader of Keynes's book not to exist except in Ramsey's mind.

It must again be repeated that R.B. Braithwaite's editorial foreword, placed at the front of the 1973 CWJMK ,Volume 8 edition by D. Moggridge, is an error of a magnitude that would measure 10 on the Richter scale for earthquakes. Moggridge's willingness to do so tells me that he understood nothing about what Keynes was putting forth in his book .Braithwaite simply regurgitates the nonsense of Ramsey as contained in the 1922 and 1926 reviews.

A very severe problem is that there is a very great probability that anyone reading this foreword will just take it for granted that, as far as providing an overall theory of probability, Keynes had already been proven to be wrong .Such a reader will already have decided, before even reading any part of the A Treatise on Probability, that Keynes got the big picture all wrong. Unfortunately, the 1973 edition has replaced the original 1921 edition when it comes to citing the A Treatise on Probability. The result is that all readers of the 1973 edition of Keynes's A Treatise on Probability have been completely mislead and misinformed about the nature of Keynes's logical theory of probability long BEFORE they have even started to read the first chapter ,which involved Keynes's beginning, initial application of Boole's relational, propositional logic ,as presented by Boole in 1854 in his The Laws of Thought (LT) in chapters I, XI ,and XII.

Bateman's comment above is representative of someone who writes about Keynes's A Treatise on Probability and writes about Keynes's logical theory of probability, but who has read only tiny bits and pieces of Keynes's work Bateman's major sources for his assessments do not come from Keynes's TP, but are based on Ramsey's and Braithwaite's error filled work .Bateman has substituted Ramsey and Braithwaite for Keynes. His assumption appears to be that Ramsey was a genius ,so he had to have been right about what he was claiming/asserting about Keynes's theory ,even though it is obvious that Ramsey is making up definitions that do not exist in the TP .It turns out that Ramsey has no understanding of Keynes's use of Boole's relational ,propositional logic .If he had ,then he would have immediately rejected his own claim that such propositions do not exist.

What is fundamental in Keynes's work is not discussed anywhere in any of Bateman's contributions on Keynes's logical theory of probability over the last 35 years.

Keynes basic, fundamental topics are

- The Boolean foundations of Keynes's relational ,propositional logic
- The Boolean foundations of Keynes's interval valued approach to probability
- The Boolean connection to Keynes's POI
- The Boolean foundations of Keynes's mathematical theory of induction
- Keynes's finite probabilities

- Keynes's conventional coefficient of risk and weight ,c ,that transforms linear ,additive, numerical probabilities into non linear , non additive ,non numerical probabilities or decision weights

- Keynes's w index to measure the evidential weight of the argument ,V

- Keynes's safety first approach based on Chebyshev's Inequality

- Keynes's application of inverse probability

- Keynes's extension of his relational ,propositional logic to propositional functions, so as to build a logical foundation for statistics

Bateman shows no awareness of the ten fundamental results accomplished by Keynes in his book. However, Bateman did read a lot of what other academicians had written about Keynes's A Treatise on Probability (1921,TP) ,with Ramsey's and Brathwaite's work being singled out for major attention by Bateman.

Bateman's latest work ,published in the December ,2021 issue *of The Journal of the History of Economic Thought* ,blindly asserts that Ramsey's critiques of Keynes were correct. He also claims that Keynes "capitulated" (this appears to be Bateman's favorite conclusion regarding how Keynes was supposed to have reacted to Ramsey's claims) to Ramsey and became an advocate of either the subjective theory of Ramsey or the inter subjective theory postulated to have been created by Keynes by D. Gillies (2000) .The Keynes-Townshend correspondence of 1937-38 shows that there is not a shred of support for either of the ruminations /musings of Bateman or Gillies (2000). Keynes simply tells Townshend (1937-1938) that the General Theory is based on Keynes's logical theory of probability as presented in the TP. No mention is made of F P Ramsey. No mention is made of subjective probability. No mention is made of inter subjective probability.

Consider the following assessment made by Bateman in 2016 about Ramsey's 1922 review:

"Ramsey's most noticed achievement (to date) by historians of economic thought is probably the open challenge he made to Keynes's Treatise on Probability, a criticism that first appeared during the second ten-week term of Ramsey's second year as an undergraduate in a short review published in the Cambridge Magazine (January 1922). Keynes had postulated that probability is an objective logical relation between two propositions; Ramsey denied that any such relations existed, completely undercutting the work that had taken Keynes roughly fifteen years to bring to fruition in 1921.

How did an eighteen-year-old undergraduate have the audacity to make such a critique and how had he been able to publish it? This is the story that Paul tells so well. "(Bateman, 2016, p.182).

Let us take a look at this alleged "open challenge...to Keynes's Treatise on Probability", which Bateman claims that Ramsey made in his 1922 *Cambridge Magazine* review , a review which Bertrand Russell (1922, 1948, 1959) refuted with a simple counter example in a small footnote in his July, 1922 review in *the Mathematical Gazette* on p.120.It is

indeed truly strange that Bateman could have overlooked Russell's clear and straightforward refutation of Ramsey on p.120 while at the same time citing from pages 119 and 121 of Russell's review in his contributions in journals inBateman,1987 and Bateman , 1990:

"First, he thinks that between any two non-self-contradictory propositions there holds a probability relation (Axiom I), for example between 'My carpet is blue' and 'Napoleon was a great general'; it is easily seen that it leads to contradictions to assign the probability 1/2 to such cases, and Mr. Keynes would conclude that the probability is not numerical. But it would seem that in such cases there is no probability; that, for a logical relation, other than a truth function, to hold between two propositions, there must be some connection between them. If this be so, there is no such probability as the probability that' my carpet is blue' given only that 'Napoleon was a great general', and there is therefore no question of assigning a numerical value."(Ramsey, 1922, p.3).

Ramsey's opening paragraph is completely and totally erroneous.

First, there is no such axiom I anywhere in Keynes's A Treatise on Probability. Second, Keynes's definitions of his argument form and relevance -irrelevance logic on pp.4-6 and 52-26, respectively, completely rule out Ramsey's bizarre assessment using his example.

Third, Ramsey's "my carpet is blue' given only that 'Napoleon was a great general" example was easily refuted by Russell with his similar Napoleon example, specifically designed by Russell to demonstrate the utterly preposterous nature of Ramsey's example, in July, 1922. The only honorable option for Ramsey to do in 1922 was to publish a complete and total retraction of his January, 1922 review .Instead, he kept repeating the same type of example in private papers that culminated in his disastrous 1926 contribution, which we shall cover later. A simple comparison of Ramsey's example with Russell's example leads to the complete collapse of Ramsey's critique:

"* I do not know whether Mr. Keynes has considered and rejected a definition of irrelevance which, prima facie, would be simpler than his. He does not state definitely whether every pair of propositions has some probability-relation, but I think he does not hold this view. I think he would say, e.g., that there is no probability-relation between the propositions ' 2+2=4 ' and ' Napoleon disliked poodles.' If so, it would seem natural to define h as irrelevant to a when a/h does not exist."(Russell, 1922, p.120,star footnote).

Thus, Ramsey's 'My carpet is blue' and 'Napoleon was a great general' example and Russell's counter example,' 2+2=4 ' and ' Napoleon disliked poodles', lead to the conclusion that Ramsey's assessment of Keynes's propositional ,relational logic, which is built on Boole, is ,using Franklin's accurate description ,"rubbish."(Academia.edu. Discussion - Comment, 2021) Consider the following claim made by Ramsey in 1926 .This example is the foundation of Bateman's 36 years of work on Keynes's logical theory of probability in which he constantly claims that Keynes rejected his own theory in 1931 based on the overwhelming critique published by Ramsey in 1922 and 1926:

"Mr. Keynes starts from the supposition that we make probable inferences for which we claim objective validity; we proceed from full belief in one proposition to partial belief in another, and we claim that this procedure is objectively right, so that if another man in similar circumstances entertained a different degree of belief, he would be wrong in doing so. Mr. Keynes accounts for this by supposing that between any two propositions, taken as premiss and conclusion, there holds one and only one relation of a certain sort called probability relations; and that if, in any given case, the relation is that of degree α, from full belief in the premiss, we should, if we were rational, proceed to a belief of degree α in the conclusion."(Ramsey, 1926 [1931]; In Kyburg and Smokler, 1980 (2nd ed.), pp.26-27; Bateman, 1987, p.106).

Of course, all of these claims involve Ramsey in severe error.

First, Keynes doesn't start from a supposition. He starts by adapting the content ,first presented by Boole in 1854 in chapters I, XI, and XII of *The Laws of Thought,* as the foundation for his approach to probability. Keynes is not assuming anything here. It is Ramsey who is showing that he is ignorant of Boole's achievement, which was built upon by J. N Keynes, L. Wittgenstein, B. Russell, G. E. Moore and W E Johnson, to mention just a few names. Second, nowhere does Keynes claim objective validity, which can only occur in a deductive logic .Third, nowhere does Keynes claim that his procedure is objectively right. Keynes claims that it is rational because it is based on all of the relevant, available evidence which appears in the h propositions. Ramsey is confused by the word rational. Fourth, the circumstances, as well as the skillset and degree of optimism -pessimism of the decision maker, must be identical, not similar. Fifth, Keynes never claimed that a decision maker would be wrong. Keynes stated that such a decision maker wound be non-rational, as opposed to being irrational. Thus, for Keynes, the only rational answer for both of the Ellsberg two urn ball (red and black ball) problems is that the probability of a red or black ball being drawn is ½, based on a correct application of Keynes's POI, as based on Keynes's summary on pp.52-56 of the *A Treatise on Probability* (TP;1921) ,and not on Keynes's restatement of the erroneous Laplace-Bernoulli version on p. 42. Sixth, Keynes's relational propositional logic does not hold' between any two propositions', but only between sets of propositions that are related, associated ,relevant or connected by a degree of similarity (or likeness or resemblance). Ramsey's claim above is simply a regurgitation in a slightly altered and rewritten form of the false claim he made in 1922 in his Jan., Cambridge Magazine review on p.3 that "between any two non contradictory propositions there holds a logical relation", which Ramsey then illustrated with his nonsensical 'my carpet is blue; Napoleon was a great general 'example .Neither of these propositions are related or connected .All of Ramsey's examples involve unconnected or unrelated propositions. Seventh, there holds one and only one relation of logical probability for exact, precise, numerically connected probabilities based on the POI or for the calculation of a least upper bound (LUB) and /or greatest lower bound (GLB)for Keynes's interval valued probabilities. Eighth, Keynes's approach does not hold 'in any given case'; it holds only in some given cases.

Nineth, Ramsey is very confused in his belief that Keynes's approach only holds for two propositions, i.e. for one h proposition and one a proposition .In fact, Keynes works with sets of h and a propositions. The best example of this would be his Darwin example on pp.5-6 and its extension on p.160-161(1921) of the TP.

Bateman relies blindly on Ramsey's badly mistaken and confused caricature of Keynes's propositional, relational logic ,as given by Ramsey above ,as being a correct summary of Keynes's theory .Bateman's position is obviously wrong and has been wrong for 35 years .The question that must be raised here is whether Bateman ever actually read *Keynes' A Treatise on Probability* .Bateman appears to have simply assumed that Ramsey had read it ,which he had not. What Ramsey had done was to read perhaps 15 pages or so, which he then proceeded to cobble together in a puzzling manner .Bateman then proceeded to interpret this puzzling concoction in a confusing and confused manner .What is really puzzling is how this concoction could have been accepted by the vast, overwhelming majority of academicians for 100 years as providing evidence proving that Keynes's theory was badly flawed .

It is not surprising that Bateman would cite Misak's very similar, error filled assessments of Keynes' logical theory of probability to 'prove ' that his much earlier ,error filled assessments of Keynes's book were sound. In fact, Misak, just like Bateman, was completely blinded to the erroneous nature of both of Ramsey's 1922 and 1926 reviews by her belief that Ramsey was a genius .She simply accepted as true statements that are obviously false ,such as"…between any two non contradictory propositions there holds a logical relation".

How anyone could believe that Keynes's theory would encompass conditional probability assessments ,such as what is the conditional probability that my carpet is green, given that Napoleon was a great general or what is the conditional probability That this is red, given that that is round ,is bewildering .Again, one has to raise the question whether Misak ever read any part of Keynes's book, such as the first 6 pages .That is all that it would have taken for Misak to seriously have started to question Ramsey's assessments.

I can find no evidence supporting a conclusion that B. Bateman ever read Keynes's entire *A Treatise on Probability* beyond some bits and pieces of pages selected in a random fashion. There is no coherent understanding of the role played by Boole. None of the important points made by Broad (1922), Edgeworth (1922) or Russell (1922) in their reviews of Keynes's 1921 book are contained in any of Bateman's work. There are simply no citations made to Keynes's TP in his articles involving any of the major ,fundamental breakthroughs listed in my abstract. Bateman skips what Russell called Keynes's mathematical theory of induction involving the initial ,a priori probability,p_0 ,and the relationships between p_0 and the all important finite probabilities ,ϵ and η, which are numerical probabilities that are greater than numerical or non numerical (interval) probabilities (Keynes, 1921, pp.233-239;253-257).

Bateman's research program was to read carefully <u>what other</u> academicians had written about Keynes's logical theory of

probability; unfortunately, Bateman, like Misak, failed to realize that these other so called 'Keynes scholars'(for instance, Braithwaite (1973), Good (1962), Gillies (2000), Hacking (1980), Mellor (1983, 1995), Methven (2015),Monk (1991), Wheeler (2012), Zabell (1991, 2005), etc.) had not read the TP themselves. They had read Ramsey.

We will use as an example Bertrand Russell's truly scholarly and learned review ,which ,together with the 1922 reviews of Broad and Edgeworth in *Mind* and *The Journal of the Royal Statistical Society*, are vastly superior to the work of Misak , Misak's so called 'Keynes scholars' or the Post Keynesian 'Keynesian fundamentalists', such as Skidelsky (1992), Runde (1994) ,Davis(1994,202) ,etc., all put together. Basically, Keynes scholarship is a complete disaster, as it is all based on Ramsey's two reviews while Russell, Edgeworth and Broad are simply ignored.

Misak and Bateman both claimed to have read Russell's July, 1922 review and both cited Russell's review .However, there is a giant blind spot in their work. They never cited Russell's obvious refutation of Ramsey that was provided by Russell on page 120 of his 1922 review .Now no one else ,who has written on Keynes's TP ,has cited this page either. It is a very glaring omission because ,once one understood what Russell had done ,it leads to only one conclusion, which is that Ramsey did not know what he was talking about .This means that it was always the case that the Keynes Discontinuity Hypothesis was utter nonsense from the beginning .

So it is with Bateman, Misak, the Keynes scholars and the Keynesian Fundamentalists. All of their work on Keynes's TP since 1975 is an intellectual waste of time and effort that is serving to block any possibility of work being done that would add to the superior work produced by Russell, Edgeworth and Broad plus Hailperin on Keynes's logical theory of probability.

7. DAVIS AND THE KEYNES DISCONTINUITY HYPOTHESIS

Consider the following statement from B. Davis:

"… because how people rationally understand the world is always limited. Second, Keynes denied– when we can form probability judgments – that all probability relations must be susceptible of numerical representation. Some are qualitative in nature – we think something is more probable than something else but cannot say in what degree – and yet are still rational.

For Keynes, this was a matter of how one made probabilistic decisions when uncertainty interfered with doing so in a still reasonable if not fully logical way. Consider again his emphasis on non-numerical probabilities. They fall short by the standard of Bayesian thinking but that conception presupposes we do not encounter them and only live in small worlds. Thus, when we acknowledge that we also live in large worlds, Bayesians have nothing to offer, though non-numerical, qualitative probability judgments often provide us reliable means for making decisions. Indeed, frequently it is sufficient to simply know something is more or less probable to determine a course of action."(Davis, 2021)

Similarly, Davis does not see the contradiction that is involved in his view, that "Some are qualitative in nature – we think something is more probable than something else but cannot say in what degree – and yet are still rational." Of course, Keynes would tell Davis that he was not rational if he were to act on his belief that "we think something is more probable than something else but cannot say in what degree- and yet are still rational."

Davis suffers from a severe logical illusion. A rational probability must come in degrees .Davis has severely confused Keynes's interval valued, 'non numerical 'probabilities', with ordinal probability. Ordinal probabilities are only accepted by Keynes as being a 'practical' application of his theory if and only if they are conditional probabilities; however, they are not rational degrees of probability as ordinal probability does not come in degrees.

Consider that all of Davis's assertions about Keynes abandoning his logical theory of probability are based on one source, Frank P. Ramsey:

"Past debate over such a "break" focussed upon Keynes's not unambiguous assertion in 1930 that Frank Ramsey had indeed been correct in his critique of the main philosophical ideas of the Treatise…A supporting argument for his position occurs in his judgment on the debate over Keynes's response to Ramsey, where he repeats his conclusion that "the 'beliefs' concerned related to the period before substantive work on the fellowship dissertation," and agrees with Anna Carabelli (1988) that "Keynes 'accepted none of the main points which are characteristic of the view of probability advanced by Ramsey in his 1926 article' " (p. 623). There are, however, a number of difficulties with this view of Keynes's development. First, Keynes bases his logical account of probability in the Treatise on the idea that we employ intuition to directly grasp (indefinable) probability relations, but is then explicit in "My Early Beliefs" in saying that one of the most objectionable intellectual practices that he and his early friends relied upon was to claim to exercise a "direct unanalysable intuition about which it was useless and impossible to argue" (Keynes, 1971-89, X, p. 437). This "neo-platonism," as he termed it, "combined a dogmatic treatment as to the nature of experience with a method of handling it which was extravagantly scholastic" (Ibid ., p. 438). Second, and relatedly, after Ramsey had criticized the idea that we intuit probability relations (' I do not perceive them, and ... moreover I shrewdly suspect that others do nor perceive them either" [Ramsey, 1978]), Keynes replied, "I think he is right" (Keynes, 1971-89, X, pp. 338-39). Accordingly, though Keynes may well not have adhered to any of the subjective probability ideas Ramsey espoused, as Moggridge concludes following Carabelli, he clearly agreed that Ramsey was correct in an important criticism of Keynes's own views on the Treatise.

Together, these two points indicate that the Treatise was not free of criticism in Keynes's eyes, at least after 1930. "(Davis, 1994, pp.361-362).

Nowhere in Keynes 's review does Keynes state what Davis claims he does .Pace Davis, Keynes's "I think he is right " refers to Ramsey's purely mathematical demonstration that the mathematical laws of the probability calculus ,which are

based on additivity and linearity ,can be derived from the axioms of his subjective approach as regards degrees of belief. Ramsey's numerical probabilities have nothing to do with Keynes 's interval valued probabilities, which are non additive and non linear and DO NOT satisfy the purely mathematical laws of the probability calculus.

Davis, like all other Post Keynesian, Institutionalists and heterodox economists writing on Keynes's TP, has no idea about what Keynes was doing in the TP. The major reason for this current state of affairs is that they have no comprehension of what one can call the Boole-Keynes connection, where Keynes builds on Boole's interval valued theory of probability.

Hishiyama, whose 1969 article correctly diagnosed what the problem was, which was that the A Treatise on Probability was never read, has only two English citations in the literature since 1969.Hishiyama's article is not cited because he opened up a can of worms for both orthodox and heterodox economists with his correct diagnosis about the failure of all economists to (a) master Keynes's new (Boolean) logic and (b) to grasp the connections between Keynes's TP and General Theory. Hishiyama's diagnosis still holds in 2022.

Finally, all Post Keynesian, Institutionalists and Heterodox economists have severely erred in their acceptance of the myth of the 18 year old boy genius, who supposedly arrived at Cambridge University, England in 1921 and showed Keynes in 1922, in his January, Cambridge Magazine review that his logical theory of probability rested on an objective, logical, probability relation between propositions that did not exist.

It is then claimed that Keynes later capitulated to Ramsey in 1931 and repudiated his own theory, after which he based his *General Theory* on Ramsey's approach. This myth is still very widely accepted in academia in 2022. It is a myth once it is realized that Russell actually refuted all of Ramsey's claims in one, small footnote contained on page 120 of Russell's July, 1922 review of Keynes's book for the *Mathematical Gazette*. Until this myth is completely rejected, it is simply impossible for any progress to be made about the close connections which exist between Keynes's TP and Keynes's GT.

9. CONCLUSION

Like Ramsey and Braithwaite before them, Carabelli, O'Donnell, Runde, Bateman and Davis have absolutely no understanding of what is entailed by Keynes's Boolean, relational, propositional logic. Carabelli, O'Donnell, Runde, Bateman and Davis have all made the exact same errors as Ramsey and Braithwaite did before them as regards Keynes's application of his relevance -irrelevance logic that makes all of Ramsey's objections intellectually worthless.

Keynes's new, logical framework, first recognized by Hishiyama, was Boole's original relational, propositional logic that Keynes extended and developed in his analysis of Part II of the TP. At no point in his life did Ramsey ever have a clue that Keynes was building on Boole. Neither do Carabelli, O'Donnell, Runde, Bateman and Davis.

Keynes rejected Ramsey's additive theory of subjective probability out of hand, except as a very, special case, because the existence of uncertainty requires, as a necessary condition, the condition of non additivity. If there is no non additive probability, then there is no uncertainty, only risk. Ramsey's theory can only deal with risk.

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