A Comparison of Emerging and Developed Economy Portfolio Performance Under COVID-19

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Abstract:

Orientation: The performance of three different portfolio allocation strategies is assessed in a developed and a developing economy during different economic conditions over a period of seven years.

Research purpose: Evaluate the performance of the portfolios – namely, the tangent, minimum-variance, and maximally diversified portfolio – across a developed and a developing economy and investigate the advantages and disadvantages that each portfolio poses in differing economic conditions.

Motivation for the study: Understanding the benefits and drawbacks of each of these portfolios in times of crisis and in times of economic expansion could assist asset managers in making effectiveallocation decisions for their portfolios in different economic conditions.

Research approach/design and method: Portfolio optimisation under various constraints.

Main findings: Tangent portfolios produced superior returns to the other portfolios and the US portfolios consistently outperformed the South African ones. The minimum variance portfolio provided greater returns and downside protection than the maximally diversified portfolio during the COVID-19 market crash for the developed economy, while the opposite was observed for the developing economy.

Practical/managerial implications: Practical knowledge of how each of the portfolios perform within different economic climates can assist asset managers to produce positive performance in times of recession and expansion.

Contribution/value-add: Information and analysis on each of these portfolio asset allocation strategies during various economic conditions assists asset managers in finding the most effective way to structure their portfolios.

Keywords: COVID-19, portfolio performance, developing economy.

1. INTRODUCTION

Markowitz's (1952, 1959) work, which formed the bedrock of modern investment theory, is now commonly grouped and known as Modern Portfolio Theory (MPT).MPT is effectively a framework for the assembly of investment portfolios whose expected returns are maximised, while their risk is simultaneously minimised (Mangram, 2013). An integral part of MPT is to reduce the overall risk component of an investment portfolio through the concept of diversification, whereby investment securities are carefully selected and weighted together in a manner that will reduce the overall risk of the portfolio to be lower than any individual asset (Mangram, 2013).

Markowitz (1959) outlines how a "good" investment portfolio is more than merely a large list of shares and bonds, but rather a balance of integrated investments built to suit the needs of the investor, offering both opportunities of great potential return as well as protection against downside risk. The choice of assets in a portfolio differs for different investors and is dependent on the investor's appetite for risk and return, but two objectives that remain constant among investors is:

- they desire high returns, and
- those returns should be subject to little uncertainty (Markowitz, 1959).

The Efficient Frontier is an essential component to MPT and the construction of investment portfolios as different combinations of assets in a portfolio produce differing levels of risk and return and it is the Efficient Frontier set out by Markowitz (1952) that displays the "best" of these combinations. The concept of "best" is different for each investor, but in this sense these combinations of securities are considered the "best" because they are the most efficient as the portfolios that plot along Markowitz's (1952) Efficient Frontier are those that produce the highest level of excess return (i.e. returns above the risk-free rate) for a given level of risk (Tracey, 2020). The portfolio that yields the greatest return

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Table 1. Constituents of South African Portfolio.

Naspers	E-commerce and fintech	British American Tobacco	Tobacco
BHP Billiton		Richemont	Luxury goods
Anglo American	Mining	Standard Bank	Banking
Anglogold Ashanti		Nedbank	
Goldfields		ABSA	

does not mean that it is the portfolio with the minimum variance, or least uncertainty, as there is a trade-off between risk and return and investors require to be compensated with greater return for taking on greater levels of risk or reduce their levels of risk by foregoing extra return (Markowitz, 1952 and Jorion, 1992).

The Minimum-Variance (MV) and Tangent (TG) portfolios are portfolios that plot along the efficient frontier and according to Markowitz's (1952) portfolio selection theory – these portfolios are diversified, as their variance cannot – at the same level of expected return – bereduced any further (Jorion, 1992, 2003). The Maximum Diversification Portfolio (MDP) is a portfolio that challenges Markowitz's (1952) efficient set as it aims to provide alternative portfolios that have higher return levels and lower risk levels, compared to the traditional efficient set MV and TG portfolios, by maximising a system of measurement known as the Diversification Ratio (Choueifaty, Froidure, & Reynier, 2012).

Using securities selected from a developing economy (South Africa) and securities selected from a developed economy (USA), this paper takes a deeper look into the performance of the minimum-variance, tangent, and maximally diversified portfolios over a period from 2011 to 2021 across these two economies. Weights and returns for each portfolio were calculated and their fluctuations were analysed over time, this paper looks to investigate whether there is a certain portfolio that displays a clear investment advantage, in the developed or developing economy, as well as looking into which portfolio performed best and provided the greatest protection amid the COVID-19 pandemic.

A brief background of salient points is provided in Section 2, followed by a review of the relevant literature in Section 3. Section 4 discusses the data and methodology used to generate the results. Section 5 discusses the results, and Section 6 concludes.

2. BACKGROUND

This paper investigates the performance of the MV, TG and MDP portfolios from 2011 to 2021in two different economies, namely South Africa and the USA. The securities that were selected were the top ten market capitalisation stocks in South Africa and the top ten market capitalisation stocks in the US. The reason for choosing the ten largest market cap stocks in each economy is because due to their high market capitalisation, they are frequently traded and are therefore the most liquid stocks of the last 10 years that can be examined. The period that is being examined is from January 2011 until January 2021 – this period was chosen as it includes

both the economic recovery after the Global Financial Crisis, as well as the crash caused by the COVID-19 pandemic beginning in March 2020. The shares selected to represent the South African and US situations were chosen to allow the examination of the performance level of each portfolio (MV, TG and MDP) in developed and developing economies.

The top ten market capitalisation stocks chosen for the South African portfolios were as shown in Table **1**.

For most of the period under examination (January 2011 – January 2021), South Africa was under the Zuma administration. Zuma's time in power (2009-2017) in South Africa was plagued by corruption and political turmoil that has had lasting negative effects on the South African economy (Hamill, 2020). Zuma's term brought about huge damage to the economy with majority of the country's financial statistics experiencing a massive decline as the growth rate in SA from 2011-2017 barely rose above 1.5% per annum (Business Tech, 2019). This stagnant economic growth was highlighted by increasing unemployment, substantial increases in public debt levels, decrease in annual GDP per capita from US$8066 (2011) to US$6268 (2017), net SA Foreign Direct Investment as a percentage of GDP fell from +22.7% (2010) to -29% (2017), average annual returns on the Johannesburg Stock Exchange between 2009 and 2019 have finished either last or 2nd last when compared with MSCI Europe, MSCI Japan, the MSCI World Index and the S&P500(Business Tech, 2019).

Ramaphosa came to power in 2018, but recovering from the damage that took place under Zuma was always going to be a difficult task and in the build up to the pandemic none of South Africa's crucial economic indicators had improved with the country formally entering a recession after two quarters of negative growth by early 2020 (Hamill, 2020). In the same period, there was considerable financial outflow with foreign investors selling nearly USD6bn worth of equities and bonds. This culminated with Moody's, S&P and Fitch downgrading SA's debt to sub investment grade (Hamill, 2020).

The top ten market capitalisation stocks that used to create the US portfolios are shown in Table **2**.

The US economy is the dominant global economy and was at an historical high in the build up to the pandemic. The Dow Jones had reached record highs and unemployment was at the lowest levels experienced for 50 years (BBC News, 2020). Post the Global Financial Crisis of 2008/09, the US economy recovered significantly well and continued to grow, consistently, year on year. Fig. (**1**) shows the growth of each

Table 2. Constituents of South African Portfolio.

Apple	Information technology	Visa	Electronic payments
Microsoft		Mastercard	
Amazon	e-commerce	Walmart	Retail
Berkshire Hathaway	Insurance/investments	Disney	Entertainment
JP Morgan	Banking	Johnson & Johnson	Pharmaceuticals
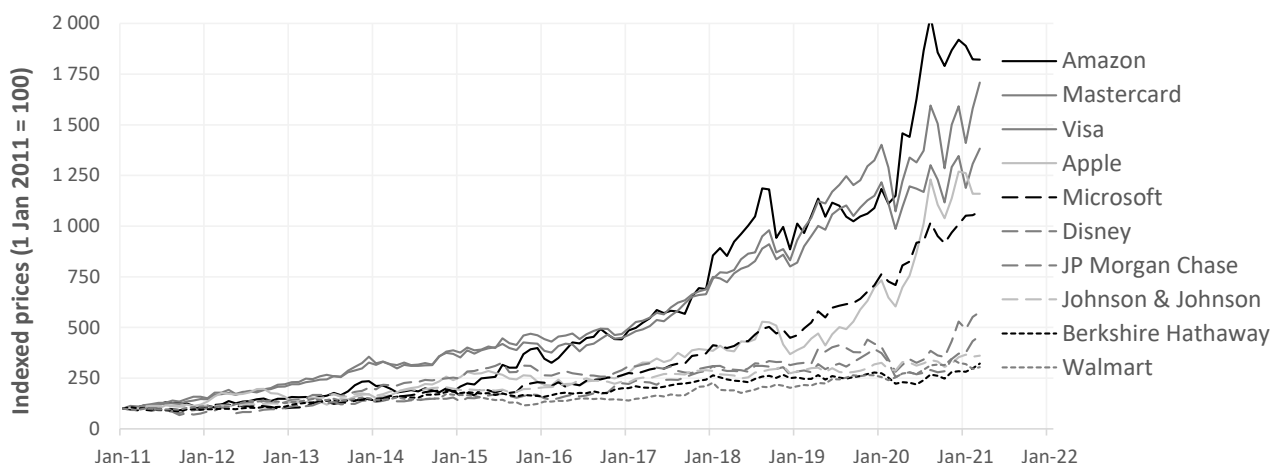
Fig. (1). Growth of US stocks rebased to 100 in January 2011. Legend arranged by rank order. Source: Authors' calculations.

of the 10 US shares from 2011-2021 rebased to 100 in January 2011.

In March 2020, the US economy recorded the largest GDP shock in history because of the COVID-19 pandemic –nearly four years of economic growth were wiped out. This malaise was felt worldwide, but the US financial market showed remarkable resilience and recovered back to pre-pandemic levels by the end of 2020 (BBC News, 2020).

3. LITERATURE REVIEW

Since Markowitz (1952) established the foundation for MPT, the issue of portfolio diversification has been at the forefront of financial debate. Markowitz's (1959) work that developed the MPT was later added to by Sharpe (1964) who developed the theory of the Capital Asset Pricing Model (CAPM), which values securities as a function of undiversifiable risk. This advances the idea of the capital market line (CML) and the efficient frontier (Mangram, 2013). CAPM theory demonstrates that the point of tangency between the CML and the efficient frontier (the tangent portfolio) is the 'most efficient' portfolio because, all else equal, it generates the highest level of expected excess return (above the risk-free rate) relative to the risk taken to generate that return (Sharpe, 1964).

Empirical studies have shown that the MV portfolio can generate returns that are greater than broad market cap-weighted indexes, while simultaneously maintaining lower levels of volatility displaying the inefficiency of the market cap-weighted index. This has been recently explored and reconfirmed by Ao, Mengmeng, Yingying & Zheng (2019).

Furthermore, the introduction of the MDP portfolio – and the concept of the diversification ratio – was also shown to be an efficient alternative to the market cap-weighted index (Choueifaty, Froidure, & Reynier, 2012).

3.1. Efficient Frontier

The concept behind the efficient frontier relies on a few assumptions that do not necessarily represent reality. The Efficient Frontier is graphically displayed as a curve that maximises return for a given level of risk. The portfolios that plot along the curve are said to be efficient as they are the lowest level of risk that can be taken on for a desired level of expected return – portfolios that plot below the curve are suboptimal, or inefficient, as they do not provide sufficient levels of return for their given levels of risk (Dikov, 2020). The greater the covariance in the combination of assets selected leads to a smaller standard deviation (less risk) within a portfolio, thus displaying one of the major implications of the efficient frontier – the benefits of diversification (Mangram, 2013). Markowitz's (1959) theory implies that the rational investor, one who is seeking the greatest possible return for the smallest level of risk, should always invest along the efficient set to avoid inefficiency. Both the MV and the TG portfolios plot along the efficient frontier seen in Fig. (**2**).

3.2. Minimum-Variance Portfolio

The minimum variance portfolio is located on the furthest left point of the efficient frontier, at a point where the level of absolute risk is at a minimum. The points on the efficient frontier in Fig. (**2**) that plot below the MV portfolio are considered sub-optimal as they carry both higher levels of risk
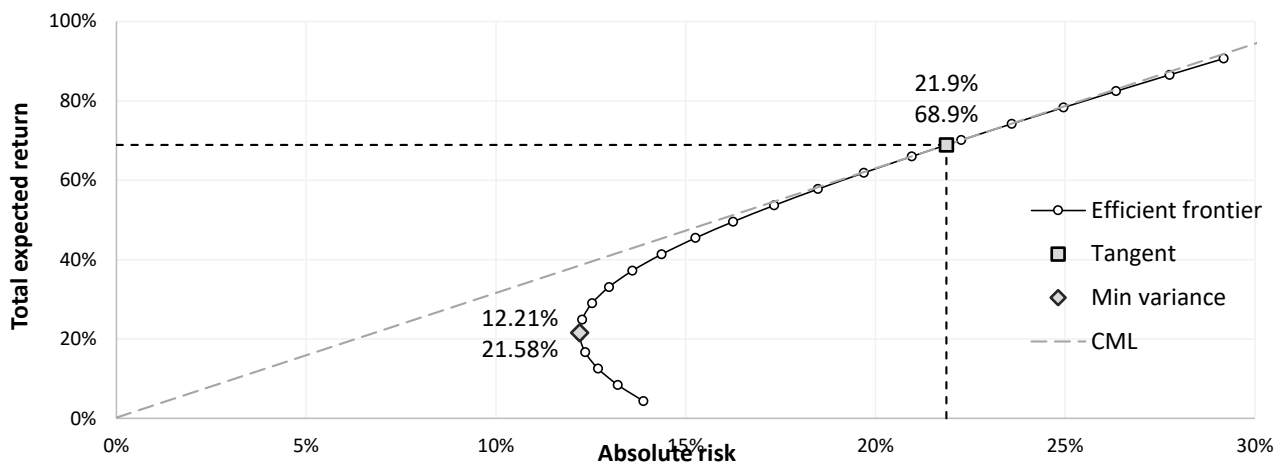
Fig. (2). Efficient frontier for US stocks in February 2021. Source: Author calculations.

with less return. The MV portfolio is unique as its optimal security weights are dependent on the securities' covariance matrix without considering the expected returns (Reh, Krüger, & Liesenfeld, 2022). The single-index model asserts that the optimal security weights for the MV portfolio depend far more on beta-related risk rather than unsystematic risk which is with the intuition of optimal portfolio diversification (Clarke, Silva & Thorley, 2011). The portion of risk that can be attributed to market exposure (undiversifiable systematic risk) can be derived analytically under the single-index model; it is equal to the ratio of portfolio β to the long-only β (Menchero & Hu, 2006). The ratio is stable and displays nearly 90% of risk in the long-only MV portfolio is systematic (Reh, Krüger, & Liesenfeld, 2022). Empirical results showed that the MV portfolio had surprisingly strong average return performance, while the optimal weight of securities showed that minimising variance in the general mean-variance optimisation functions was adequate to exclude many investable securities from the portfolio (Jorion, 2003).

3.3. Tangent Portfolio

Markowitz's (1952) work on mean-variance optimisation was the first of its kind and it laid the foundations of MPT. Under the mean-variance approach, portfolio selection is based off maximising return for every given level of risk that creating a set of efficient portfolios – all with combinations of minimum risk and maximum return – where investors are able choose a portfolio that suits their preferences/risk appetite. All these portfolios are efficient, but there is one portfolio that maximises return per one unit of risk – essentially the most efficient portfolio – this is the TG portfolio (Bilir, 2016), the point of tangency of the CML and the efficient frontier, where the CML slope is the Sharpe ratio (Larsen, & Resnick, 2001 and Menchero & Hu, 2006).

To find the TG portfolio the Sharpe ratio needs to be maximised through dividing the risk premium by the portfolio standard deviation. Bilir (2016) created a hypothetical portfolio comprising ten equally weighted stocks that were traded on the Istanbul Stock Exchange and used Markowitz's mean-variance model to find the optimal portfolios (Jorion, 2003). There were various portfolios that were constructed

and the portfolio that maximised the Sharpe ratio (TG portfolio) was found to be the most efficient in terms of the amount of return per one unit of risk. The TG portfolio exhibited close to three times more return than the original equally weighted portfolio (Bilir, 2016).

3.4. Maximum Diversification Portfolio

The Maximum Diversification Portfolio (MDP), introduced by Choueifaty (2006), defined a metric known as the diversification ratio (DR). Choueifaty (2006) showed that a portfolio that maximises the diversification ratio is the maximally diverse portfolio of a selection of stocks and may be considered as an efficient alternative to the market capitalisation-weighted index. Choueifaty (2006) defined the DR as the quotient of a portfolio's average volatility and its overall volatility. This measure embraces diversification because the volatility of a long-only portfolio is either less than or equal to the weighted sum of the portfolio's overall volatility (Choueifaty, Froidure, & Reynier, 2012).

The diversification ratio of a portfolio is the measure of diversification that is gained from holding securities that do not have a perfect correlation and, intuitively, portfolios that consist of securities with high correlation figures will not be well diversified and possess a very low diversification ratio (Kone, 2021).

Choueifaty, Froidure & Reynier (2012) define the *core property* of the Maximum Diversification Portfolio to help provide an intuitive understanding of the nature of the MDP. Any stock held by the MDP is less correlated to the MDP than any of the stocks that comprise it and all stocks which comprise the MDP have the identical correlation with it. The MDP therefore effectively represents all securities in a considered universe, even if the portfolio does not actually hold all these securities.

The empirical results of Choueifaty and Coignard (2008) provided a mathematical definition for diversification that showed the maximally diversified portfolios have higher Sharpe ratios, lower volatilities, and higher returns in the long run than market capitalisation indexes, thus exhibiting the value of the theoretical framework for diversification. Their results exhibited that the MDP could provide sound

alternatives to the traditional MV portfolio, market cap-weighted index and the equally weighted portfolio, although it is difficult to determine whether the MDP plotted along the efficient frontier (Choueifaty & Coignard, 2008).

4. DATA AND METHODOLOGY

Data used were the monthly price data of the top 10 market cap stocks in the US and South Africa over the period from 2011 until 2021. Price data were collected from Bloomberg and all weightings, returns, risk measures, diversification ratios, efficient frontiers and portfolios were created and calculated on Excel. The top 10 market cap stocks were chosen as these stocks are the most frequently traded and are therefore the most liquid stocks that can be examined for the period under investigation (2011 – 2021). The efficient frontiers were created and calculated using the top 10 market cap stocks from each economy and the US 3m Treasury rate and the SA 3m JIBAR rate as proxies for the risk-free rate for the US and South Africa, respectively. The reason for looking at the performance of the MV, TG and MD portfolios within the context of the USA and South Africa respectively is to be able to compare the performance of the portfolios within a developed and a developing economy. The period from 2011 – 2021 was selected because within this period the financial recovery from 2008 Global Financial Crisis can be examined, as well as the market crash and the effects of the COVID-19 pandemic within the context of a developed and developing economy.

4.1. Minimum Variance Portfolio

MV portfolio weights are calculated using:

$$w_{MV} = \frac{\Omega^{-1}\mathbf{1}}{\mathbf{1}'\Omega^{-1}\mathbf{1}} \tag{1}$$

where w_{MV} are the MV portfolio weights, Ω^{-1} is the inverse of the $n \times n$ variance covariance matrix of relevant asset returns, $\mathbf{1}$ is a $n \times 1$ vector of 1s (Jorion, 2003).

4.2. Tangent Portfolio

For TG portfolios excluding a risk-free asset, the unconstrained weights are:

$$w_{TG} = \frac{\Omega^{-1}E}{E'\Omega^{-1}\mathbf{1}} \tag{2}$$

where $\mathbf{w}_{TG}$ is the vector of TG portfolio weights, and E is the $n \times 1$ vector of the asset's expected returns. For portfolios with a risk-free asset with return r_f, the portfolio weights are:

$$w_{TG} = \frac{\Omega^{-1}(E - r_f \cdot \mathbf{1})}{\mathbf{1}'\Omega^{-1}(E - r_f \cdot \mathbf{1})} \tag{3}$$

The weights of the *unconstrained* TG portfolio requires an analytical, iterative solution (Jorion, 2003).

4.3. Maximum Diversification Portfolio

The MD portfolio may be optimised using the following Lagrange function (Pemberton & Rau, 2007):

$$L(w, \lambda) = \frac{1}{2}w'\Omega w - \lambda(w'\sigma - 1)$$

where w represents the matrix of portfolio weights to be optimised, λ is a Lagrange multiplier, Ω is the $n \times n$ variance covariance matrix of returns, and σ is the $n \times 1$ vector of asset volatilities:

$$\sigma = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \cdot \\ \sigma_N \end{bmatrix}$$

L is the Lagrangian.

Some linear algebra shows that

$$w_{MD} = \lambda\Omega^{-1}\sigma \tag{4}$$

$\sigma' w_{MD} = 1$ so   multiplying   (4)   by σ'   gives $\sigma' w_{MD} = \lambda\sigma'\Omega^{-1}\sigma = 1$, so

$$\lambda = \frac{1}{\sigma'\Omega^{-1}\sigma} \tag{5}$$

Substituting into (5) into (4) gives:

$$w_{MD} = \frac{\Omega^{-1}\sigma}{\sigma'\Omega^{-1}\sigma} \tag{6}$$

Using (6) and with $\sum_{i=1}^{n} w_{MDi} = 1$, the diversification ratio, D, is defined as:

$$D = \frac{w'_{MD}\Sigma}{\sqrt{w'_{MD}\Omega w_{MD}}} \tag{7}$$

D comprises a volatility-weighted correlation ($\rho(w_{MD})$) and a volatility-weighted concentration, $C(w_{MD})$

$$D = \frac{1}{\sqrt{\rho(w_{MD})(1 - C(w_{MD})) + C(w_{MD})}}$$

where $\rho(w_{MD})$ is the portfolio's volatility-weighted asset correlation:

$$\rho(w_{MD}) = \frac{\sum_{i \neq j}(w_i\sigma_i w_j\sigma_j)\rho_{i,j}}{\sum_{i \neq j}(w_i\sigma_i w_j\sigma_j)} \tag{8}$$

$C(w_{MD})$ is the portfolio's volatility-weighted concentration ratio:

Table 3. Weight Calculation Summary for the Three Approaches.

Minimum variance	$w_{MV} = \dfrac{\Omega^{-1}\mathbf{1}}{\mathbf{1}'\Omega^{-1}\mathbf{1}}$	1
Tangent	$w_{TG} = \dfrac{\Omega^{-1}(E - r_f \cdot \mathbf{1})}{\mathbf{1}'\Omega^{-1}(E - r_f \cdot \mathbf{1})}$	3
Maximum diversification	$w_{MD} = \dfrac{\Omega^{-1}\sigma}{\sigma'\Omega^{-1}\sigma}$	6

$$C(w_{MD}) = \frac{\sum_i (w_i \sigma_i)^2}{(\sum_i w_i \sigma_i)^2} \tag{9}$$

C measures the concentration of portfolio *weights*, as well as the risk concentration (Choueifaty, Froidure & Reynier, 2011).

Table **3** summarises the weight formulae for the three approaches.

5. RESULTS AND DISCUSSION

5.1. Performance of Global Markets (2011 – 2021)

The efficient frontiers were created using data from 2011 – 2021, but three years of data were needed to be able to create the efficient frontiers – therefore the data were rolled on a three-year basis so the results that are being evaluated are from 2014 – 2021.

In 2011, the global markets were still in recovery post the Global Financial Crisis (GFC) of 2008, but by 2014 the global markets had recovered back to above to pre-GFC levels. There was a period of steady growth from 2014 until the beginning of 2020 when the world went into lockdown due to the COVID-19 pandemic and the markets experienced a considerable crash. The recovery post pandemic was far quicker, compared to the recovery post-GFC that took several years, and took just under a year to return to pre-pandemic levels.

5.1.1. South African Markets (2011 – 2021)

The JSE All Share Index (ALSI) had recovered back to pre-GFC levels by 2011 and continued to steadily grow at a similar rate to the US stock market. By 2014, the ALSI was outperforming the Dow Jones Industrial Average before the growth started to plateau and the South African markets experienced relatively steady growth reaching a pre-pandemic high at the beginning of 2018 – during this period between 2014 and 2018, the South African TG portfolio showed superior returns to the MV and MD portfolios albeit at far higher levels of risk. By 2020, the ALSI was not at the pre-pandemic high of 2018 and after the outbreak of COVID-19, the market crashed significantly wiping out all the growth it had experienced since 2014. The post-pandemic recovery was swift with the ALSI recovering back to pre-pandemic levels within a year of the crash and reaching an all-time high by March of 2021. All three of the South African TG, MV and MD portfolios experienced increased levels of risk after the outbreak of the pandemic – the most dramatic increase in risk was felt by the TG portfolio, but this was coupled with a significant increase in returns between 2020 and 2021 as the market recovered, while the MV and MD portfolio returns briefly fell into the negatives.

5.1.2. US Markets (2011 – 2021)

The severity of the crash of the GFC was felt far worse for the Dow Jones Industrial Average as compared to the ALSI and it took until 2013 for the Dow Jones Industrial Average to reach pre-GFC levels. Between 2011 and 2016, the ALSI and the Dow Jones Industrial average were at similar levels, and both grew steadily until the US markets entered a boom near the beginning of 2017. Between 2017 and 2019, the US TG portfolio had a considerable spike in returns as well as an increase in risk levels over this period, while the US MV and MD portfolios experienced far lower levels of return compared to the TG portfolio – these portfolios were able to maintain far more stable returns at significantly lower levels of risk. From 2017 – 2020, the Dow Jones Industrial Average substantially outperformed the ALSI and the US markets had reached record highs by March of 2020. As was the case with the ALSI, the outbreak of the COVID-19 pandemic caused an immense crash for the Dow Jones Industrial Average, but the recovery was near immediate and by March of 2021 the US markets had reached a new all-time high. The TG and the MD portfolios experienced an increase in risk level after the market crash –with the TG portfolio experiencing the more dramatic increase in the level of risk, but this was coupled with extremely high returns between 2020 and 2021. The MV portfolio maintained a steady level of risk and return throughout the period under examination.

5.2. Minimum-Variance Portfolio

The MV portfolio focuses on minimising the standard deviation of the overall portfolio while simultaneously achieving returns for the investor. The MV portfolio is attained by combining and weighting the securities in a portfolio in a certain way to reduce the correlation of the securities in the portfolio and, in turn, minimising the price volatility of the overall portfolio. The MV portfolio is an appropriate investment technique for pension funds or for investors who are
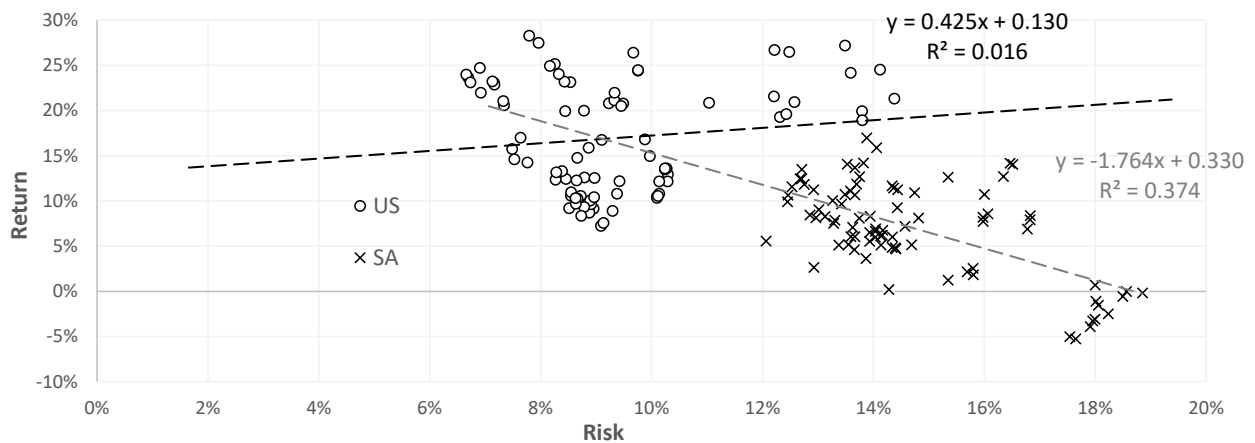
Fig. (3). Scatter plot displaying comparison of risk and return of the minimum-variance portfolio for both the US and South Africa.
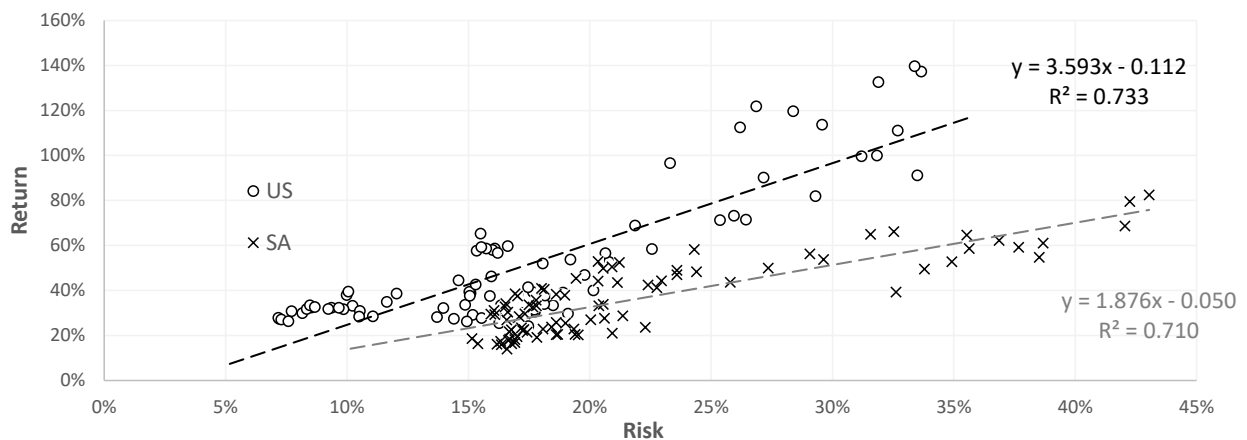

Fig. (4). Scatter plot displaying comparison of risk and return of the tangent portfolio for both the US and South Africa.

risk-averse and are looking for slow and steady returns over time while also guarding against major losses in a market crash. Fig. (**3**) displays a comparison of the risk and return relationship for the US and South African MV portfolios.

Fig. (**3**) shows an upward sloping trendline for the US MV portfolio which is what is expected as investors are compensated with extra return for taking on additional levels of risk. The results from Fig. (**3**) also illustrate that the US MV portfolio was able to achieve greater returns at lower levels of risk than that of the SA MV portfolio – with the US MV returns ranging from between 7.25% to 28.29% and levels of risk ranging from 6.66% to 14.38%, while the SA MV had lower returns varying from -5.26% to 17% and higher levels of risk that varied between 12.06% to 18.85% – this was no surprise as the US economy is far more developed, while the shares in the developing South African economy are expected to be associated with higher risk levels. Fig. (**3**) displays a downward sloping trendline for the SA MV portfolio which represents decreasing returns with increasing levels of risk, this could be explained by the fact that South Africa is adeveloping economy and the riskier shares in this developing environment seem to be worse than the less risky shares.

5.3. Tangent Portfolio

The TG portfolio aims to maximise a metric known as the Sharpe Ratio where an investor can attain the maximum pos-

sible return per unit of risk. The TG portfolio is more appropriate for investors with a higher risk tolerance that are looking to achieve superior returns as the TG portfolio aims to gain greater returns at higher levels of risk as compared to the MV and MD portfolios.

Fig. (**4**) depicts the risk and return relationship for the US and SA TG portfolios. The returns for both the US and SA TG portfolios are far superior to those attained by the MV or MD portfolios, but this was achieved at higher levels of risk. The US TG portfolio outperformed the SA TG portfolio in the period under examination, and as can be seen in Figure 4, the US portfolio experienced higher levels of return at almost every level of risk. This can also be seen by looking at the trendlines of each portfolio in Fig. (**4**) – both trendlines are upward sloping, which intuitively would make sense as investors are compensated with greater return for taking on additional units of risk – but the slope of the US portfolio's trendline is nearly two times as steep as the slope of the South African portfolio's trendline, which translates to double the return for the US portfolio compared to the SA portfolio for every additional unit of risk. Between 2014 and 2021, the US TG portfolio had returns that ranged from 24.16% to 139.59% and it had levels of risk that ranged from 7.17% to 33.66%, compared to the SA TG portfolio that had returns ranging from 13.86% to 82.42% with levels of risk between 15.15% to 43.05%. The US economy experienced more significant growth than the SA economy between 2014
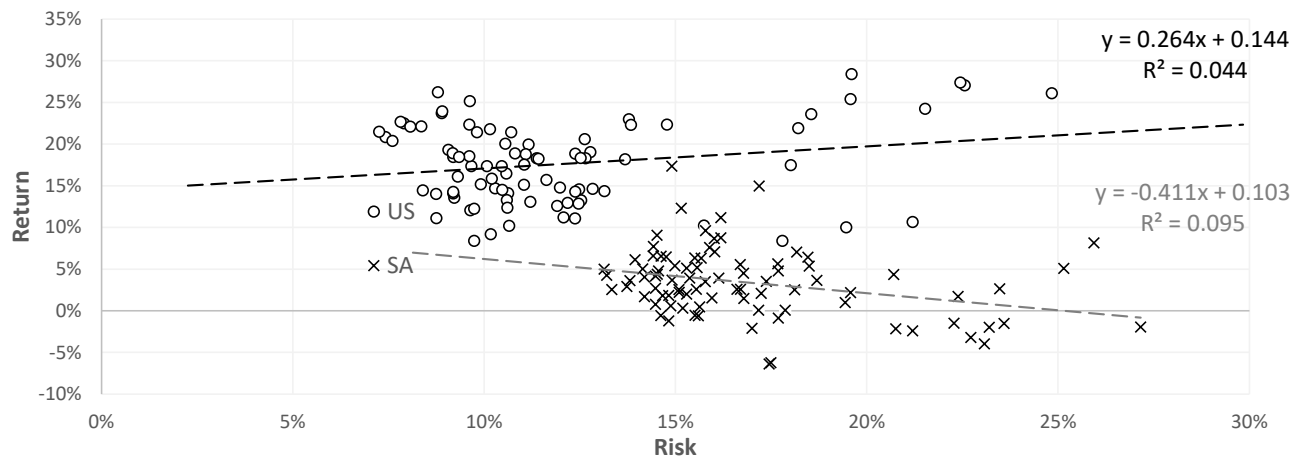
Fig. (5). Scatter plot displaying comparison of risk and return of the maximum diversification portfolio for both the US and South Africa.
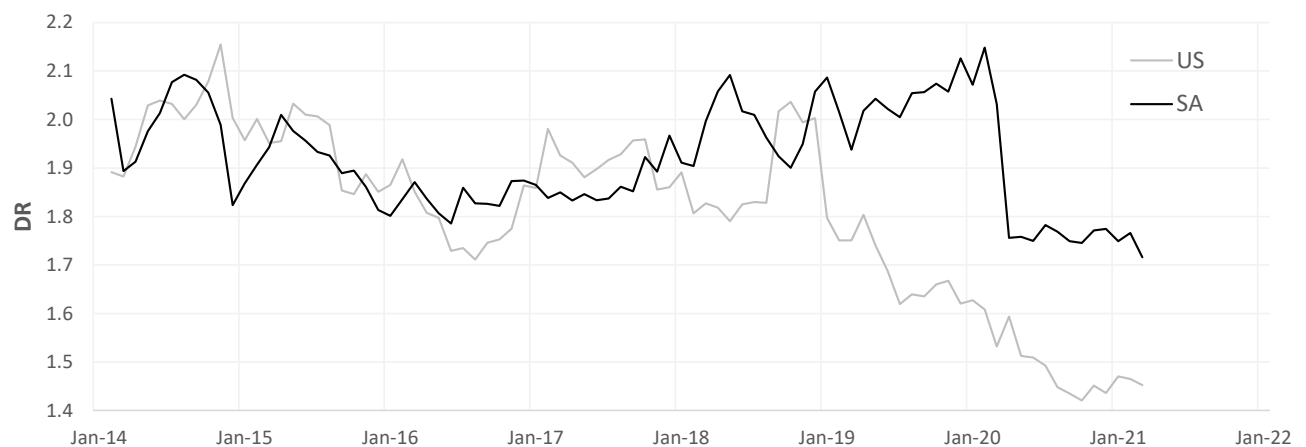

Fig. (6). Comparison of diversification ratios between US and South African portfolios.

and 2021 and this could attribute to the superior returns achieved by the US TG portfolio compared to the SA TG portfolio, while the higher levels of risk associated with the South African portfolio would be expected for a developing economy that is not as established as the US economy. As expected, the returns for the TG portfolios are considerably higher than that of the MV or MD portfolios for each economy.

5.4. Maximum Diversification Portfolio

MD portfolios displayed similar results to that of the MV portfolios. The MD portfolio attempts to diversify away as much risk as possible, so the return is expected to be lower than traditionally riskier portfolios, such as the TG portfolio, and closer to that of the MV portfolio. Fig. (**5**) is a scatter plot that is displaying the relationship between risk and return for both the SA and US MD portfolios. The US trendline in Fig. (**5**) is upward sloping whereas the SA trendline is downward sloping, which is like the result that was observed for the MV portfolios. It is interesting to note that US trendline for the MV portfolio in Fig. (**3**) is 1.6 times steeper than the US trendline observed in Fig. (**5**) – showing there is more reward for additional units of risk for the developed economy using a minimum-variance portfolio opposed to a maximally diversified portfolio – whereas the SA trendline for the

MV portfolio in Fig. (**3**) is 4.29 times negatively steeper than the trendline for the SA MD portfolio in Fig. (**5**). This implies that the SA MV portfolio would lose 4.29 times more return than the SA MD portfolio for each additional unit of risk, therefore there is no reward in taking on additional units of risk for a minimum-variance or a maximally diversified portfolio in a developing economy. Between 2014 and 2021, the US MD portfolio had returns that varied between 8.37% and 28.38%, while the returns for the SA MD portfolio fluctuated from -6.39% to 17.36%. For the same period, the US and SA MD portfolios experienced risk levels that varied from 7.26% to 24.84% and 13.14% to 27.16%, respectively.

The MD portfolio focuses on creating a maximally diversified portfolio by maximising a metric known as the Diversification Ratio (DR). The DR is calculated as a ratio of the weighted average of volatilities by the overall portfolio volatility and when a portfolio of stocks maximises the DR, that combination of securities is said to be maximally diversified. A portfolio that is maximally diversified should provide protection against downside risk, while also attempting to provide superior returns to traditional portfolios. Fig. (**6**) compares the performance of the SA and US MD portfolios as a function of their diversification ratios over the observation period, where a higher DR implies more benefits from diversification.
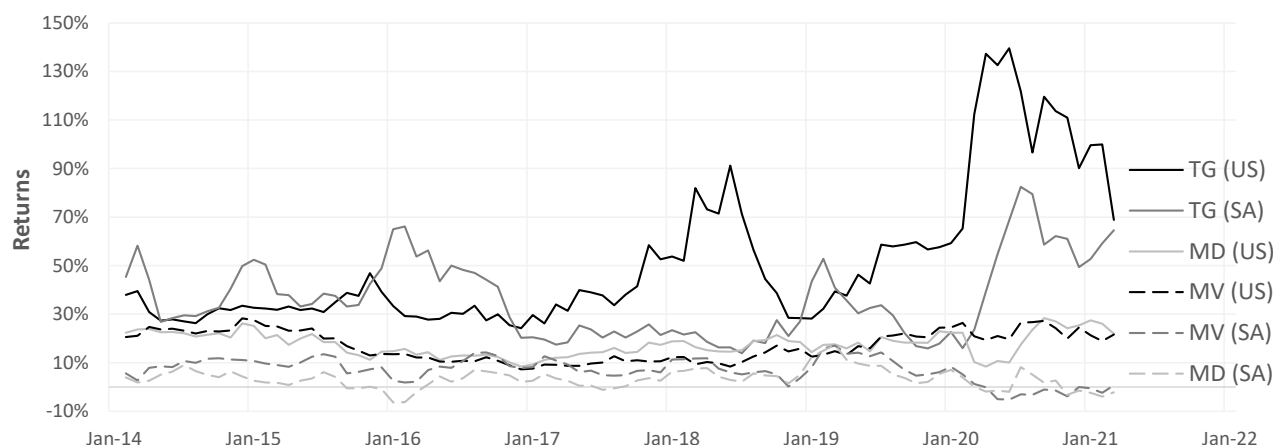
Fig. (7). Comparison of returns between US and South African portfolios for each of the minimum-variance, tangent and maximum diversification portfolios.

The DRs for both the SA and US MD portfolios reached a high of 2.15 for the period under examination, with US portfolio reaching its high point near the end of 2014 whereas the SA portfolio's DR reached its peak in the early stages of 2020. The SA portfolio slightly outperformed the US portfolio in terms of their DRs as the SA portfolio averaged a DR of 1.92 compared to the US portfolio's average of 1.81 for the observation period.

The ratios for the two economies tracked each other closely between 2014 and 2019, with minor disparities near the end of 2014 when the US portfolio reached its peak and another separation close to the beginning of 2018 where the SA portfolio was slightly outperforming the US portfolio. The period after 2019 is where the first major discrepancies are observed as the US DR began to decrease quite significantly, falling from 2.00 in December of 2018 to its low point of 1.42 in mid-2020. The decrease in the DR for the US portfolio coincided with a boom for the US stock market during 2019, where there was a movement towards tech stocks with tech companies such as Apple and Microsoft experiencing gains that helped them reach trillion-dollar status. In contrast to this, the SA portfolio's DR remained above 2.00 for the majority of 2019 and reached a high of 2.15 in February 2020 before falling drastically to 1.76 by April of 2020 – this fall in DR corresponds with the market crash in early 2020 due to the outbreak of the COVID-19 pandemic.

5.5. MV, TG, MD Performance Comparison (2014 – 2021)

For the period under examination, each of the US portfolios displayed superior returns compared to their South African counterparts. The TG portfolio was by far the best performing portfolio for both economies with the US TG portfolio averaging returns of 51.28% for the observation period compared to the SA TG portfolio's average of 36.28%. The US MD portfolio slightly outperformed the US MV portfolio with average returns of 17.59% and 16.97%, respectively. The opposite was true for the South African MD and MV portfolios as the SA MV portfolio experienced an average return of 7.01% that was more than double that of the SA MD portfolio that showed an average return of only 3.37%. When comparing the equivalent portfolios across the two

economies, the US portfolios all experienced greater returns compared to the SA portfolios with the most significant difference in the MD portfolios as the SA MD portfolio displayed the lowest absolute returns, as well as the biggest relative difference compared to the US MD portfolio that averaged returns 5.22 times greater than the SA MD portfolio. The US MV achieved average returns 2.42 times greater than the SA MV and the US TG had an average return that was 1.41 times greater than the SA TG portfolio.

Fig. (7) shows the returns of all three portfolios for each economy from 2014 to 2021. Both the TG portfolios had the most significant returns and both portfolios experienced dramatic increases in returns at beginning of 2020 during the market crash when the world went into lockdown. The dramatic increase in return for the US TG portfolio in 2020 can be explained by the fact that the US portfolio consists of many tech stocks such as Microsoft, Mastercard, Apple, Visa and Amazon that performed particularly well during this period as the world was forced to move online due to the lockdown. The SA TG portfolio also experienced a major increase in return between 2020 and 2021 – not quite to the extent of the US portfolio – but this could be attributed to the swift market recovery after the crash as the South African markets recovered swiftly during this period reaching an all-time high by 2021.

The SA TG portfolio outperformed the US TG portfolio on a few brief occasions, but most significantly between January 2016 to December 2016 where the SA TG portfolio averaged a return of 47.02% compared to the US TG portfolio averaging 29.08% during this period. During 2017 and 2018, the US TG portfolio was the only portfolio to see a major increase in returns and considerably outperformed the other five portfolios reaching a pre-pandemic high of 91.15% in June of 2018, while the next best performing portfolio at that point was the SA TG portfolio sitting at just 16.34%. This increase that the US TG portfolio experienced coincided with a boom in the US stock market during 2017 where the S&P500 finished the year up by 21.7% and averaged a Sharpe Ratio of 3.2(Carlson, 2019). It is interesting to note that US TG portfolio continued to show increases in returns until its pre-pandemic high midway through 2018, although
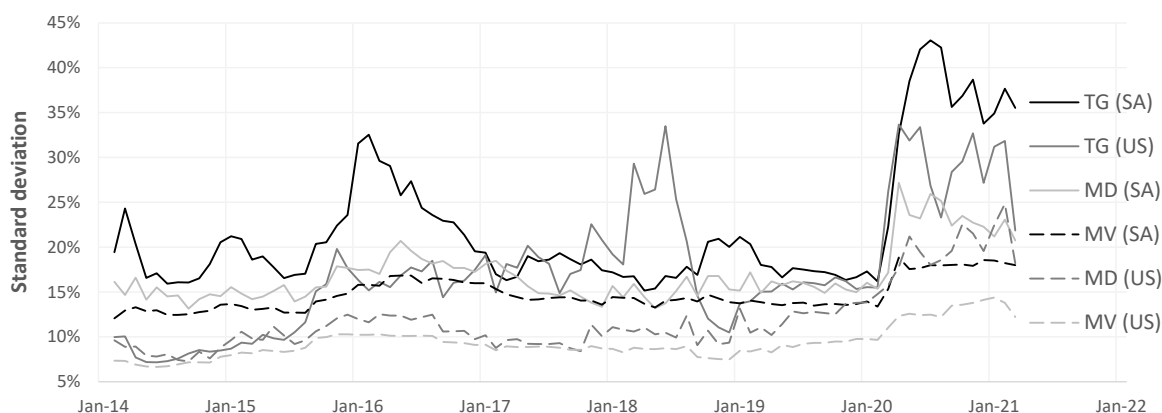
Fig. (8). Comparison of risk between US and South African portfolios for each of the minimum-variance, tangent and maximum diversification portfolios.

the US stock market struggled throughout 2018 with the S&P500 finishing the year down 4.4% (Carlson, 2019).

The performance of the MV and MD portfolios in each of the two economies followed each other closely. This is because these portfolios are essentially aiming to achieve similar goals as the minimum-variance portfolio is minimising the overall risk of a portfolio, whereas a maximally diversified portfolio attempts to reduce a portfolio's overall risk through the benefits of diversification. The returns of the US MV and MD portfolios mimicked each other's movements throughout the observation period, although the US MD consistently outperformed the US MV portfolio between 2017 and 2019. This was during the same period that the US TG portfolio saw considerable increases. In contrast, the US MV outperformed the US MD portfolio on average by 8.13% during the first half of 2020 on as the world experienced the COVID-19 induced market crash.

The SA MV portfolio consistently outperformed the SA MD portfolio for majority of the period under review. The SA MD portfolio performed relatively poorly throughout this period, falling into negative returns on multiple occasions. The only period of significance where the SA MD portfolio outperformed the SA MV was during the market crash during 2020 as the SA MD averaged returns of 1.52% while the SA MV averaged returns of -2.28% between March and October of 2020. This is a notable result as the opposite was the case for the US MV and MD portfolios. Amidst the market crash, it was the MV portfolio that achieved greater returns than the MD portfolio for the developed economy, in contrast to the developing economy where the MD portfolio outperformed the MV portfolio.

The returns attained by these portfolios were achieved at varying levels of risk. The US portfolios were able to achieve higher returns while maintaining lower levels of risk compared to their South African equivalents throughout the period that is being examined. Both TG portfolios experienced the highest risk levels, but this was to be expected as the TG portfolio undertakes increased risk to produce superior returns. The SA TG portfolio had the highest level of risk compared to the other five portfolios at an average of 22.04% for the period, while the US TG portfolio experienced an average risk level of 17.40%. The US MV portfolio

maintained the lowest level of risk for the period at an average 9.36%, while the US MD portfolio experienced an average risk level of 11.97%. Both the SA MV and MD portfolios had higher risk concentrations than the US MV and MD portfolios with the South African portfolios experiencing an average risk level of 14.71% and 16.92%, respectively. The US MV and MD portfolios performed in a conventional manner as the US MD had a slightly higher average risk concentration compared to the US MV but was compensated with a slightly higher level of average return for the period. The SA MV achieved a higher average level of return at a lower concentration of risk for the period when compared to the SA MD portfolio. The SA MD portfolio was only able to produce an average return for the period of 3.37% at an average risk level that was only 0.48% below that of the US TG portfolio at 16.92%. Fig. (**8**) displays the risk levels of all six portfolios from 2014 to 2021.

These data show that the spikes in risk levels for both the SA and US TG portfolios coincides with the periods that they experienced increases in returns. Intuitively, this makes sense because as these portfolios assumed an increase in risk, they were compensated with further return. The SA TG portfolio's risk level increased to 32.53% in February 2016 which is when the portfolio experienced its pre-pandemic high in returns of 66.08%. 2016 in South Africa was a time of major uncertainty in the country's economic policy under Zuma, and this accompanied with leading rating agencies threatening a possible downgrade of South Africa's bond rating could be a reason for the SA TG portfolio experiencing this spike in its level of risk. The US TG portfolio experienced a spike in its risk level in June 2018 to 33.50% that coincided with its pre-pandemic high in return of 91.15%. It is interesting to note that in April 2020 amidst the market crash – the US TG portfolio saw its highest level of risk throughout the period that was only 0.16% higher than the risk it experienced in June 2018, yet the returns that were achieved at that point were significantly higher, reaching 137.25%. During the 2020 market crash, the SA TG portfolio saw its peak returns as well as its highest risk level with returns amounting to 82.42% at a standard deviation of 43.05%.Yet,amidst the crisis the US TG portfolio achieved significantly superior returns at lower levels of risk when compared to its South African equivalent.
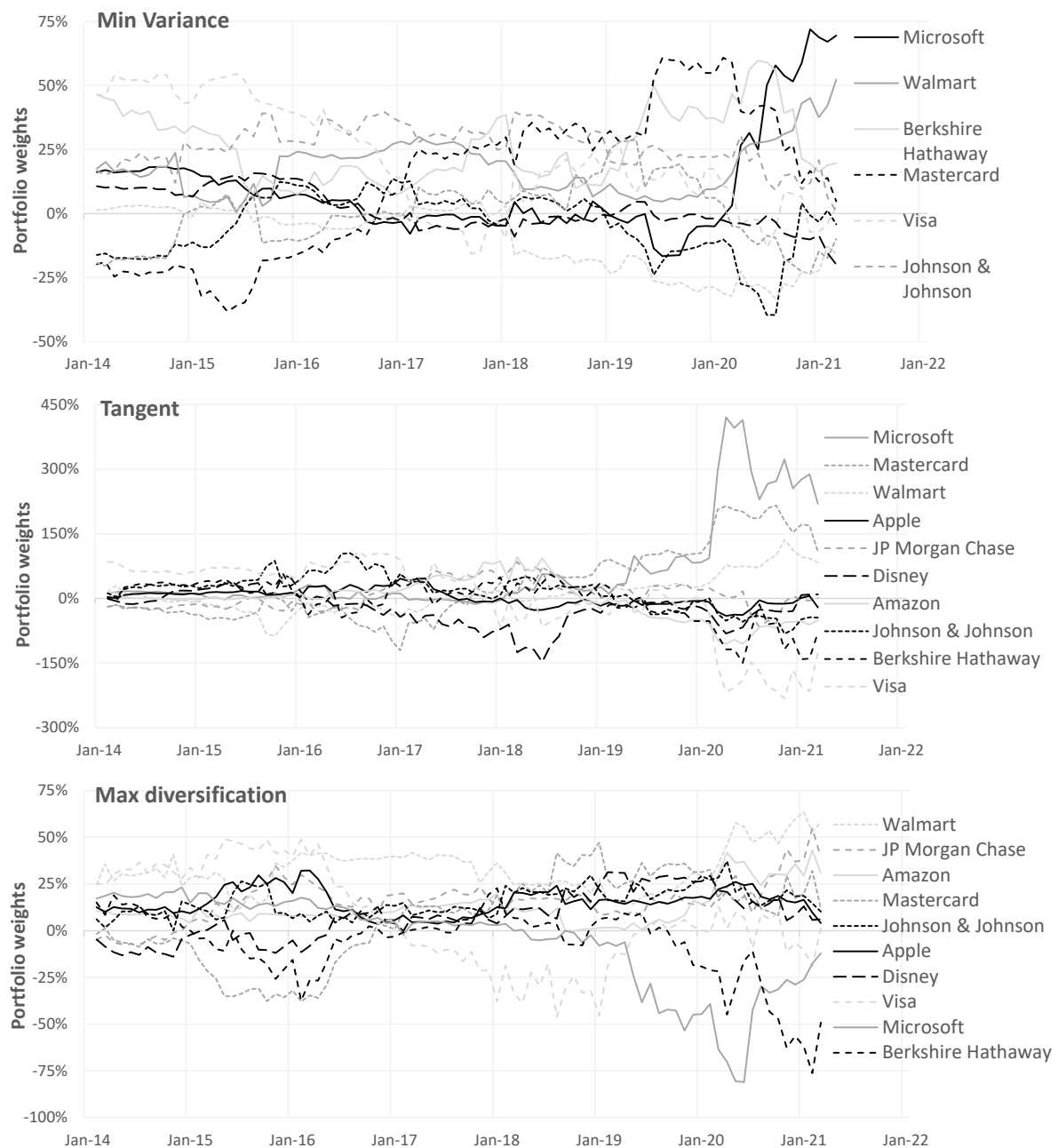
Fig. (9). Comparison of the weights for each of the US portfolios over the observation period. Identical vertical scales used for comparison.

The SA MD portfolio had higher levels of risk throughout the observation period when compared to the SA MV portfolio, and both portfolios experienced an increase in risk to their high points during the 2020 market crash to 27.16% and 18.85%, respectively. Although the SA MD portfolio saw a more significant increase in risk during the market crash, this was the only period that it produced superior returns to that of the SA MV portfolio. Similarly, the US MV portfolio maintained lower risk levels throughout the observation period when compared to the US MD portfolio. This may be the reason the US MD portfolio displayed marginally greater returns than the US MV portfolio for the period, but the significant difference in data becomes apparent post-pandemic. The US MV portfolio did not experience the dramatic spike in risk that the other five portfolios experienced

after the outbreak of the COVID-19 pandemic. From March to November of 2020, during the market crash, the US MV portfolio was able to produce an average return of 22.81% while maintaining an average level of risk of 12.66%. When compared over the same period – the US MD portfolio averaged a return of 17.77% at an average risk of 19.38%, therefore the US MV portfolio displayed it provided greater protection in a time of crisis as it was able to achieve higher returns at a lower level of risk.

5.6. Portfolio Weights

Figs. (**9** and **10**) display a comparison of the portfolio weights for the MV, TG and MD portfolios for the US and South Africa respectively.
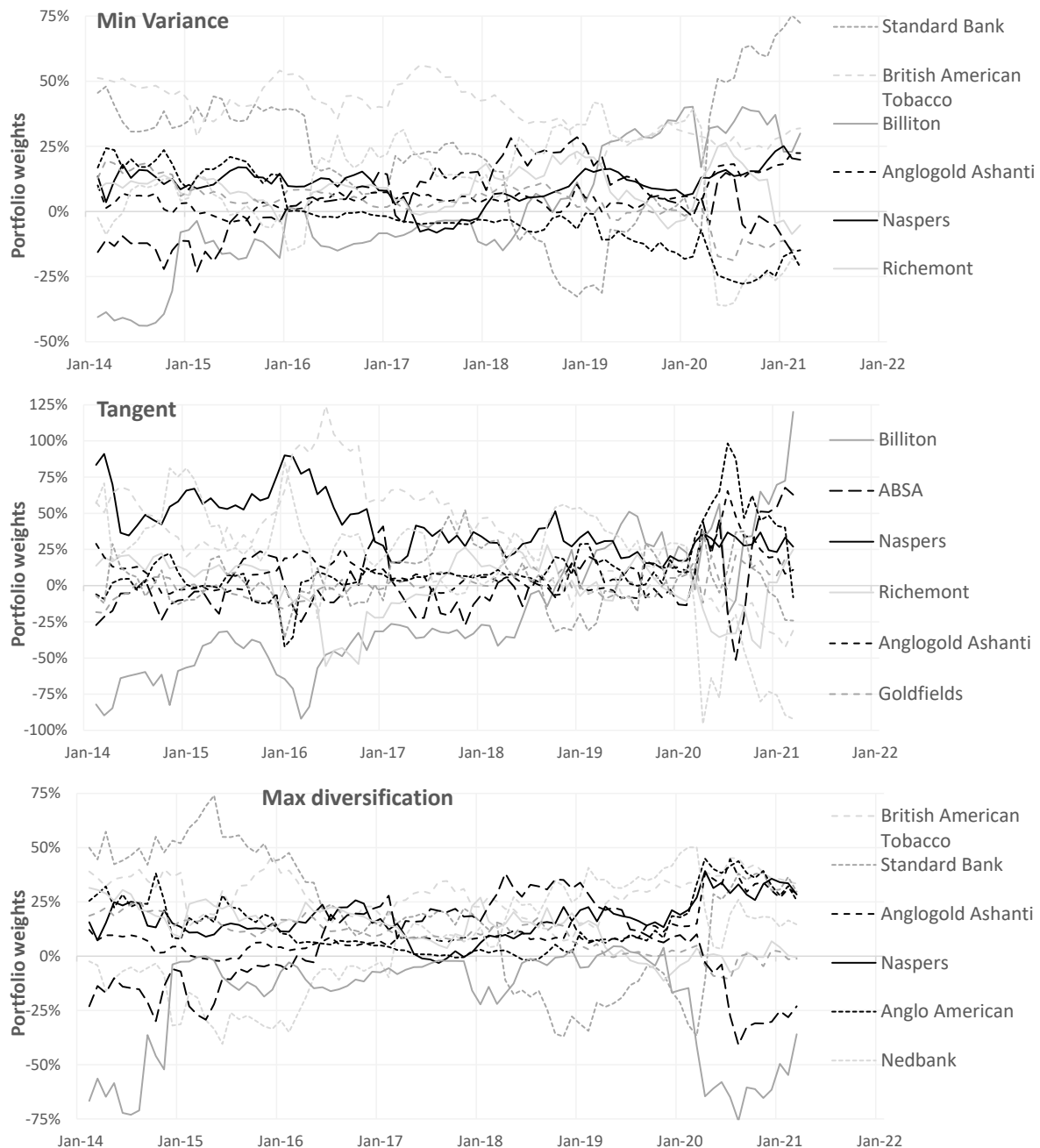
Fig. (10). Comparison of the weights for each of the SA portfolios over the observation period. Identical vertical scale used for comparison.

Each of these three portfolios that have been investigated have separate goals that they are attempting to achieve – the MV portfolio aims to minimise the overall risk of a portfolio, the TG portfolio maximises the Sharpe Ratio of a portfolio and the MD portfolio maximises the DR metric to see maximal benefits from diversification. These different objectives that are associated with each portfolio leads to differing levels of risk, return and weighting for each of them. The weights of the MV and MD portfolios are expected to be similar as they are achieving similar goals of lowering risk through different methods, while their weightings are expected to be far different to that of the riskier TG portfolio.

There is a noticeable difference in the weights of the TG portfolio compared to the MV and MD portfolios for the US

economy during the observation period. The TG portfolio makes use of leverage and short selling to a greater extent throughout the period being examined compared to the MV and MD, but the most dramatic change in weights of these portfolios is seen post-pandemic. In March of 2020 when the global markets went into disarray, the TG portfolio experienced dramatic changes in its weightings whereas the MV and MD weights remained more stable in comparison. By April 2020, amidst the worldwide lockdown, the TG portfolio was investing over 400% of the portfolioin Microsoft shares while short selling over 200% of Visa shares in the portfolio. The reason for the massive increase in investment in the Microsoft shares would be due to the exceptional returns that Microsoft experienced during the lockdown as the

world was forced to move online and countless business meetings and conferences had to be held over Microsoft Teams helping its share price skyrocket. The huge shorting of Visa shares post-pandemic for the TG portfolio could be explained by the fact that there was a major halt in transactions as the world had entered a lockdown and the vast majority of buying and selling had slowed heavily.

It is interesting to note that the percentage invested in Microsoft shares increased for both the TG and MV portfolios post-pandemic, whereas the MD portfolio was the only portfolio to short Microsoft post-pandemic. The reason for this could be because the MD portfolio is focussed on maximising the DR by weighting negatively correlated assets together, therefore as Microsoft heavily increased its returns and volatility post-pandemic, the MD would have put more weighting in shares with less return and less volatility to ensure a negative correlation between assets. It is also noteworthy that both the TG and the MV portfolios shorted Disney stocks post the outbreak of COVID-19 when one would expect investors to be heavily invested in a company like Disney during a worldwide lockdown because of their online presence. The reason for shorting Disney shares may be due to the large amount of revenue that they would have lost from the closing of Disney Parks due to the lockdown.

The South African portfolios did not experience the large disparity in weightings between the TG and the MV & MD portfolios that the US portfolios experienced post-pandemic. The immediate difference that can be noted for the South African portfolios in Figure 10 is that the TG portfolio does not experience the same drastic increase in leverage and short selling that the US TG portfolio experienced post-pandemic. Although the South African portfolios did not experience this large disparity in weights, the way the portfolios were weighted was heavily affected after the outbreak of COVID-19. Figure 10 displays the weights for each of the South African portfolios and at the point of March 2020 there is a chasm and a clear change in the way the portfolios are weighted.

In March 2020, all three portfolios increased the use of leverage and short selling, but the most considerable increase was experienced by the TG portfolio. This was also the case for the US portfolios, but to a far greater extent than what is witnessed in the South African portfolios. The SA TG and MV portfolios experienced a more distinct change in portfolio weights in March of 2020 where there is an immediate increase in use of leverage and short selling when compared to the SA MD portfolio that saw a more gradual change in its weights. Although the SA MD saw a slightly more gradual change in portfolio weights post-pandemic, it took just four months until July 2020 for the SA MD to be using short selling to a greater extent than the SA MV portfolio.

The SA MV, TG and MD portfolios were all investing in Nedbank shares for several years pre-pandemic, but post-pandemic both the TG and MV portfolios began to sell Nedbank shares with the TG portfolio selling 95.59% of the portfolio in Nedbank shares in April 2020, while the MV portfolio sold 35.84% of its portfolio in Nedbank shares in May of 2020. The MD portfolio did not short Nedbank shares post-pandemic, but between March and May of 2020 it reduced the quantity invested in Ned bank by over 45%.

The reason for the reduced investment and shorting of Nedbank shares is due to the heavily increased volatility and poor returns that Nedbank experienced post-pandemic. Nedbank experienced the lowest return of all the South African shares in March of 2020, with a return of -55.16%. It is also interesting to note that the MV and MD portfolios went from shorting Standard Bank pre-pandemic to investing a large portion of their portfolios in Standard Bank shares post-pandemic, with the MV portfolio investing over 50% and the MD investing nearly 30% of their respective portfolios in Standard Bank by May of 2020.

6. CONCLUSIONS AND SUGGESTIONS

The performance of the different portfolios varied considerably. The US portfolios all outperformed the South African equivalent portfolios with higher average levels of return and lower average levels of risk. The US TG portfolio was the best performing portfolio in terms of average returns for the period producing the greatest average return of 51.28%. The SA MD portfolio produced the lowest average returns for the period at an average of 3.37%. The alarming statistic with regards to these two portfolios is that the US TG achieved the highest return at an average risk level of 17.40%, while the SA MD portfolio achieved the lowest return at a similar average risk for the period at 16.92%. The SA MD heavily underperformed as shares were not able to be diversified as efficiently as required and therefore lacked the benefits from diversification. In the observation period, the US MV portfolio maintained the lowest level of monthly volatility, while the SA TG portfolio experienced the highest levels of volatility.

TG portfolios were the best performing portfolios in terms of returns for both economies for the observation period. Both TG portfolios displayed superior returns to the other portfolios for majority of the observation period, and both portfolios experienced considerable increases in returns amidst the COVID-19 pandemic when the global markets crashed. Although the TG portfolios generated superior returns, these were achieved at higher levels of standard deviation. This is to be expected as the tangent portfolio takes on additional risk to maximise the Sharpe Ratio and earn extra return. The US TG portfolio was able to maintain greater return and lower levels of risk when compared to the SA TG portfolio.

The MD and MV portfolios tracked each other's performance closely for each economy. The US MV portfolio maintained the lowest level of risk throughout the period. The US MD portfolio averaged a slightly higher return as well as a higher level of risk for the period when compared to the US MV portfolio. The US MV portfolio was the only portfolio that was able to maintain a stable level of risk during the COVID-19 market crash as all the other portfolios experienced spikes in their risk levels as depicted in Figure 8. Amidst the market crash post-pandemic, the US MV portfolio earned greater return at lower risk levels when compared to the US MD portfolio – displaying that the minimum-variance portfolio may provide the greatest protection in a time of crisis for a developed economy.

The SA MV portfolio maintained the lowest level of risk for the South African portfolios and still produced greater returns than the SA MD portfolio. The SA MD portfolio was

the worst performing portfolio and had returns that fell into the negatives on multiple different occasions. During the market crash of 2020, both the SA MV and MD portfolios dipped into negative returns, but this was the only period of significance where the SA MD produced greater returns than that of the SA MV portfolio albeit at higher levels of risk. This is the opposite to what was witnessed with the US portfolios, which may suggest that a maximally diversified portfolio provides greater protection amidst a time of financial crisis within a developing economy.

Although the SA MD portfolio averaged a higher DR for the period when compared to the US MD portfolio, the SA MD portfolio did not experience the benefits of diversification – future work of the MD portfolio could include more complete data from a wider universe of stocks as this will allow for more in-depth and real-world analysis that can better display diversification benefits.

Future analysis could use a larger universe of stocks to create a greater scope for diversification and improve overall portfolio performance. A larger universe of stocks in the analysis will also allow for more accurate outcomes with regards returns, diversification ratios, Sharpe Ratios and risk levels as using the top ten market cap stocks in each economy may misrepresent the results slightly.

Creating the MV, TG and MD portfolios from multiple different developed and developing economies may help identify new trends or superior portfolio strategies in different economic environments. Future analysis could also include the transaction costs of trading stocks as this will better reflect real market conditions and give investors a cost-adjusted figure for return. Incorporating more real-world conditions in the analysis will allow for more optimal and realistic portfolios to be created.

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