Optimization Model of Structural Allocation of Financial Resources in the Pension System of Ukraine

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Abstract. The article considers special conditions for the formation of the optimization model of structural allocation of financial resources in the pension system of Ukraine. A methodological approach to determining the optimal structural allocation of financial resources in the pension system is developed. The economic-mathematical model of pension provision is derived. The dependence of the optimal rate of contributions to the pension system. An algorithm for calculating the share of the capital of the pension system in the economic potential of the state using the "golden rule" of the balance of pensions is formed. Models of optimal distribution of financial resources over time between the generation of pensioners and able-bodied persons, as well as a model of parameters of state pension policy have been formed. The number of pensioners by type of pension, as well as the conditions of payment of pensions through banking institutions of Ukraine are analyzed. The coefficient of coverage of old-age pensions in Ukraine and EU countries has been determined. The coefficient of replacement of pensions by age, disability and in case of loss of a breadwinner in Ukraine and the countries of the world is modelled. The indicators of the accumulative component of the pension system of Ukraine are substantiated. It is proved that the combination of distribution and accumulative pension systems will allow investing pension funds, determining the criteria for optimizing the investment process in developing a mechanism for allocating financial resources of the pension system of Ukraine as pension savings.

Keywords: Pension System; Pension Provision; Financial Resources; Pensioners; Pension Assets; Pension Payments.
JEL Codes: G30 , H55, H75.

1. INTRODUCTION

Social protection systems cover a wide range of interrelated demographic, economic, financial, organizational and legal aspects of life through the pension system of Ukraine, which should consist of three levels (mandatory distribution, mandatory savings and voluntary savings), however, it currently operates only at the first and third levels. The structure of financial resources of the pension system as the architecture of the ratio of solidarity and accumulative levels of the pension portfolio system is formed on the basis of risk diversification, which is of fundamental importance for Ukraine.

Instead, the demographic situation of the world indicates that the main trend is an ageing population, and other problems depend on the level of development and cultural differences of each country. These changes are already affecting social security, and in the medium and long term may pose significant difficulties to existing social security systems in general and the pension system in particular. This requires a deeper study of the problems in order to combat them, and find ways to turn them into opportunities for development. The state pension programs according to the official level of salaries determine not only the amount of assigned pensions but also the fullness of the consolidated pension fund.

The priority of our study is to develop a methodological approach to determining the optimal structural allocation of financial resources in the pension system, based on methods of game theory and pension models for different levels of pensions, given the risks of an investment in investing pension assets in the economy of the state.

2. LITERATURE REVIEW

At the global level, mathematical modelling of the pension system and the creation of software products to calculate the

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effects of demographic change are developed by D. Blanchet, D. Kessler (1991), V. Brediuk, O. Dzhoshi (2013), E. Croitoru (2015), L. Mac Kellar, T. Ermolieva (1999), N. Martynenko (2017a; 2017b), V. Piskunenko (2003), O. Prykhodchenko (2011), O. Prykhodchenko (2017), V. Tropina (2017).

D. Blanchet and D. Kessler (1991) drew attention to intergenerational equity in the context of pension funding. Pay-as-you-go systems and pure-funding were considered. It was found that they have their own advantages and disadvantages, and a model was created that combines the features of both systems. V. Brediuk and O. Dzhoshi (2013) created a model of pension provision based on the example of Ukraine and showed its possibilities for a positive impact on the quality of pensioners' life. E. Croitoru (2015) drew attention to the long-term perspective of pension reforms and pointed to the fact that often their consequences cannot be predicted at a sufficient level. This phenomenon was studied on the example of Romania and the recommendations were developed for this country, though, they can be implemented in an international experience as well.

N. Martynenko (2017a; 2017b) proposed a number of ideas for improving the country's economy, with an emphasis on the pension system. The role of the shadow segment of the market and the importance of working conditions and the level of payment are shown. O. Prykhodchenko (2017) characterized the impact of the phenomenon of unofficial wages on the country's pension system. An important topic in the research by J. Pérez-Salamero González, M. Regúlez-Castillo, M. Ventura-Marco and C. Vidal-Meliá, (2022) is the relationship between mortality and various social factors, in particular, the pension system. On the example of Spain, the gradient of mortality by income was determined.

The issues of financial stability of pension systems, transition to the accumulative form of pension provision and the impact of the implemented reforms on the country's economy as a whole have been studied by V. Danich, L. Yakymova (2013), V. Kornieiev (2013), K. Kovalchuk, O. Prykhodchenko (2011), E. Lybanova (2014), V. Ponomarenko, T. Klebanovoi, N. Kyzyma, L. Yakymova, V. Danich (2012), S. Tesliuk (2016).

K. Kovalchuk, O. Prykhodchenko (2011) conducted a detailed study of the pension system of Ukraine and considered its impact on the general quality of life of citizens. In particular, the parallel of the pension system development was drawn and the dependence of the quality of life on changes in this system was clarified. E. Lybanova (2014) reviewed the existing demographic trends in Ukraine and the necessary actions to improve the situation in the country. In this context, changes in the pension system were also recommended. The role of the population living standard on the specifics of state activity was investigated by T.-Y. Pak (2020). The issue of pension provision in South Korea and its impact on the health of pensioners was covered.

3. MATERIALS AND METHODS

The decision to expand and improve the pension system leads to the construction of an effective economic-mathematical model of pension provision (EMMPP) by using the methodology (Rad, 2015), which ensures the transition from a consolidated pension scheme to a saving pension scheme in Ukraine. The construction of the EMMPP is based on the classical model of the general balance of overlapping generations (Sidelnykova, 2016), in which the number of periods is increased to provide a more flexible system when changing the constraints on the demographic structure of the population. At the same time, the social security system is mixed, i.e. it is not purely consolidated or purely saving. This approach allows realistic reflecting the period of employment of individuals and the period of their retirement.

The life of each future pensioner is divided into two periods: when he is able to work and the period of retirement. Since citizens' decisions about consumption and savings at any given time depend on their expectations of income in future, the existence of a certain balance requires that there is a clear relationship between variables at neighboring times. It should be noted that the optimal choice of consumption and savings is not linear and often cannot be obtained as an explicit function from other model variables. There are two approaches to this problem: model simplification and numerical simulation. However, both of these approaches limit the possibility of conducting a theoretical analysis of uncertainties.

Rejecting the hypothesis of the accepted assumption, we can assume that the risk of uncertainty in the model exists in the form of multiplicative stochastic shocks of the production function (Martynenko, 2017):

$$F(K, L) = uF(K, L) \qquad (1)$$

where, u – is a non-negative random variable with a certain distribution function.

We suppose that the production function has the form of the Cobb-Douglas function, that is, the interest rate and wages in equilibrium will have the same stochastic nature (Martynenko, 2017a). That is, the model maximizes the expected discounted utility, which in stochastic dynamic programming reduces a multi-period problem to a sequence of simple two-period problems. Accordingly, the Bellman equation, which characterizes the optimal intertemporal choice of the consumer, has the form (Prykhodchenko, 2011):

$$U'(ct)=\beta EtRt+1U'(ct+1) \qquad (2)$$

where, β – individual discount factor; R_{t+1} – gross interest rate between time points t and t+1.

The solution to the problem described above can be obtained explicitly only in the case of rather strict restrictions on the type of utility function or on the stochastic nature of wages and interest rates (Bonoli et al., 2005; Kovalchuk and Prykhodchenko, 2011; Shumylo, 2015).

Since the discounted income of an individual is the discounted sum of future random wages, even the strictest assumptions about the nature of the stochastic processes underlying wages wt and interest rates Rt, do not allow obtaining current consumption in the form of enough simple function of the current level of capital or output in the economy. Therefore, it is necessary to simplify the model to the case when individuals live in two periods (Bonoli et al., 2005; Kovalchuk and Prykhodchenko, 2011; Shumylo, 2015). Thus, in this model, future retirees live two periods: during the first period of life they work, and during the second - receive a pension. In addition, two generations live in the model at the same time – able-bodied and retired. At each moment of time t a new generation of future able-bodied citizens is born, the number of which Lt in 1+*n* times greater than the number of the previous generation L_{t-1}, born at the time t-1 (variable п has stood the test of time and reflects the rate of population growth). The labor market in the model is not flexible, because in the first period of their lives t individuals are offered one job, for which they receive a salary t_w and pay a contribution to the pension fund τ_t. They benefit from consumption in the first period of life c_t^1, and in the second period c_{t+1}^2. To finance their consumption in retirement, they make savings S_t in the first period of life. In the second period of life, taking into account the return on investment, they receive income $S_t R_t$, and a pension b_{t+1}. Future retirees maximize their discounted utility V_t, by considering the level of wages, interest rate and pension policy parameters τ_t and b_{t+1}set (Arza, 2006; Fehr, 2013):

$$Vt=(ct1)+\beta U(ct+12)\rightarrow max \quad (3)$$

with restrictions (Arza, 2006; Fehr, 2013):

$$ct1=wt-\tau t-st \quad (4)$$

$$ct+11=stRt+1+bt+1 \quad (5)$$

The production sector in the model is given by the Cobb-Douglas function (Arza, 2006; Fehr, 2013):

$$Y_t = F(K_tL_t) = AKat^{1-\alpha} \quad (6)$$

The supply of labor is inelastic and proportional to the number of workers L_t at the time t. The supply of capital K_t at the time t is determined by the decision on savings at the time 1-t and by the capital of the pension system, if there is such capital (Arza, 2006; Fehr, 2013):

$$K_t = s_{t-1}L_{t-1} + K_t^s \quad (7)$$

Social capital and individual savings from the total stock of capital in the economy, which is transferred by the production function. The return on social capital is thus equal to the market interest rate. The interim budgetary constraint on the pension system is given by the equation (Arza, 2006; Fehr, 2013):

$$K_{r+1}^s = L_t\tau_t - L_{t-1}b_t + R_tK_t^s \quad (8)$$

In per capita terms, the budget constraint on the pension system (9) looks like this (Arza, 2006; Fehr, 2013):

$$(1+n)k_{t+1}^s = \tau_t - \frac{1}{n+1}b_t + R_tk_t^a \quad (9)$$

This model specification allows individual savings to be negative when the social capital stock is positive. Because of the identity of all citizens in the model, aggregate savings in the economy will also be negative. In this case, all capital is supplied to the manufacturing sector by the pension system. However, such a situation is extremely unrealistic, so it is assumed that the share of savings is non-negative (Arza, 2006; Fehr, 2013; Tesliuk, 2016). Thus, it is considered that $S_t \geq 0$.

Formula (10) describes the capital market balance (Mac Kellar and Ermolieva, 1999; Ponomarenko *et al.*, 2012; Prykhodchenko, 2017):

$$(1+n)(k_{t+1} - k_t^s) = s_t(w_t, R_{t+1}, \tau_t, b_{t+1}) \quad (10)$$

Equation (10) together with the budget constraint of the pension system (9) allows obtaining the equation of the dynamics of social capital and total capital in the economy at a competitive equilibrium. Competitive equilibrium allows having in a sense any pension policy (Mac Kellar and Ermolieva, 1999; Ponomarenko *et al.*, 2012; Prykhodchenko, 2017).

Table **1** presents an algorithm for deriving the economic-mathematical model of pension provision (EMMPP), according to which the optimal contribution to the pension system is stable over time. If the saving pension scheme provides the maintenance of pensions in proportion to the average wage in the economy, then the optimal strategy will be a fixed rate of contributions to the pension system to a certain optimal level. This will allow the state economy to accumulate capital on its own.

Table 1. Derivation of Economic and Mathematical Model of Pension Provision (EMMPP).

Sphere of formation	Explanation
(1) $\tau^{PAYG} = \frac{1}{1+n} b^{PAYG}$	The model assumes that the economy is in a stationary state with a distributive pension system. In such an equilibrium, the contribution to the pension system τ^{PAYG} is determined by the required amount of pensions b^{PAYG} and population growth rate *n*.
(2) $\sum_{i=1}^{\infty} \gamma^{t-1} [U(c_t^1) + \beta U(c_{t+1}^2)] \rightarrow max$	The social optimum in the economy determines the distribution of financial resources, which is provided in central planning, taking into account the usefulness of all future generations. This maximizes the discounted amount of utility for all current and future generations.
(3) $\beta(1+n)U'(c_t^2) = \gamma U'(c_t^1), t = t_0, \ldots, \infty$ $(1+n)U'(c_t^1) = \gamma f'(k_t)U'(c_{t+1}^1),$ $t = t_0, \ldots, \infty$	First-order conditions for the central planning problem, where condition (3) determines the optimal distribution of financial resources between generations living in the same period of time, and condition (4) – the optimal distribution of financial resources over time.

(4) $U'(c_t^1) = \beta R_{t+1}\square'(c_{t+1}^2)$	After the transformations, the first-order conditions in the central planning problem (3) and (4) give the first-order condition in the optimization problem of the individual. That is, both the individual and the central planning body allocate financial resources over time efficiently. The source of inefficiency is the distribution of financial resources between generations living at the same time.
(5) $\beta(1+n)U'(c_t^2(w_{t-1}, R_t, \tau_{t-1}, b_t)) = = \gamma U'(c_t^1(w_t, R_{t+1}, \tau_t, b_{t+1}))$	Since the choice of consumption determined by an individual from the problem depends on variables $\{w_t, R_{t+1}, \tau_t, b_{t+1}\}$, the task of the pension system is to choose the parameters of the pension system $\{\tau_t, b_{t+1}\}$ such that the condition of the first order of the central planning task is fulfilled.
(6) $\beta(1+n)U'\left[c_t^2(w_{t-1}, R_t, \frac{b_t}{R_t} - \tau_{t-1})\right] = = \gamma U'\left[c_t^1(w_{t-1}, R_{t+1}, \frac{b_{t+1}}{R_{t+1}} - \tau_t)\right]$	The share of social savings in the total amount of savings increases with the increase in the contribution to the pension system τ_t. This can be seen by substituting the balance of the pension system for the equation of the balance of capital in the economy.
(7) $c_t^1 = \frac{\gamma(1-\gamma\alpha)}{\gamma+\beta} \div Ak_t^a$ (8) $c_t^2 = \frac{\beta(1-\gamma\alpha)(1+n)}{\gamma+\beta} \div Ak_t^a$	Assumptions about the logarithmic utility function allow obtaining a solution to the problem of central planning by the method of dynamic programming. The solution for the optimal consumption of workers and retirees in the problem of central planning is shown in formulas (7), (8).
(9) $c_t^{1\ ind} = \frac{1}{\gamma+\beta}(w_t - \tau_t = \frac{b_{t+1}}{R_{t+1}})$ (10) $c_t^{2\ ind} = \frac{\beta R_{t+1}}{1+\beta}(w_t - \tau_t = \frac{b_{t+1}}{R_{t+1}})$	To obtain the solution for the present value of the pension package $\frac{b_t}{R_t} - \tau/_{t-1}$, we substitute the solution of the optimization problem of the individual in the solution of the problem of central planning (7) and (8). We get the solution to the optimization problem of the citizen.
(11) $\frac{1}{1+\beta}(w_t - \tau_t = \frac{b_{t+1}}{R_{t+1}}) = \frac{\gamma(1-\gamma\alpha)}{\gamma+\beta} \times Ak_t^a$	We define a condition of optimum pension policy.
(12) $\left[\frac{\gamma(1+\beta)(1-\gamma\alpha)}{\gamma+\beta} - (1-\alpha))\right] \times Ak_t^a = \frac{b_{t+1}}{R_{t+1}} - \tau_1$	Expressing k_t+1 and R_t+1 through k_t using the balance of the physical product, we obtain the condition for the present value of the pension package.
(13) $\pi_t = \varphi\frac{\gamma}{1+n} - \frac{\gamma(1+\beta)(1-\gamma\alpha)}{(\gamma+\beta)(1-\alpha)} + 1$	The optimal contribution to the pension system π_t is determined by equation 1.

Source: built by the authors according to data.

The dependence of the optimal rate of contributions to the pension system depends on the parameters of the model (Table **2**). We determine the signs of derivatives π_t by population growth rate п, individual social discount factors β and *Y*, the parameter of the production function a and the rate of substitution ϕ. It is assumed that β, *Y* and α are positive and do not exceed one, and the substitution rate is a non-negative value. The population growth rate can have any sign, but it cannot be less than -1.

Table 2. Dependence of the Optimal Rate of Contributions to the Pension System on the Parameters of EMMPP.

Calculation Algorithm	Explanation
(1) $\frac{d\pi}{dn} = -\varphi\frac{\gamma}{(1+n)} \leq 0$	This is a completely natural result. Higher population growth rates allow for smaller contributions to the pension system with a constant amount of pension payments, as the ratio of the number of employees to the number of retirees increases. The burden on each employee related to the financing of pensions for the elderly generation is reduced.
(2) $\frac{d\pi}{dn} = \frac{\gamma}{(1+n)} \geq 0$	Relationship between variables. Higher pensions require higher contributions to the pension system.
(3) $\frac{d\pi}{dn} = \frac{\gamma(1-\gamma)(1-\alpha\gamma)}{(\gamma+\beta)^2(1-\alpha)} \geq 0$	With the growth of the individual discount factor β the usefulness of the future period in the utility function of each individual is present with more weight. On the optimal trajectory, the consumption of pensioners increases with growth β. Contributions to the pension system are increasing in order to provide a higher level of consumption for retirees.
(4) $(1+\beta)\frac{\gamma^2\alpha+2\gamma\alpha\beta-\beta}{(\gamma+\beta)^2(1-\alpha)} \geq 0$ $\geq (1+\beta)\frac{3a\beta^2-\beta}{(\gamma+\beta)^2(1-\alpha)}$	To determine the sign of the derivative, we assume that the individual discount factor β is less than the social discount factor y and that the share of capital a in total output is between 0.5 and 1.

(5) $\frac{d\pi}{dn} = (1+\beta)\frac{\gamma(\gamma-1)}{(\gamma+\beta)(1-\alpha)^2}$	When the share of capital a increases, the production function becomes closer to linear. There is a redistribution of output between the factors of production in favor of capital. At the same time, the consumption of the working generation is falling faster than the condition of optimal distribution suggests. To compensate for the decline in workers' consumption, the rate of contributions to the pension system should decrease with increasing a.

Source: built by the authors according to data.

Table **3** shows the calculation of the algorithm that determines the share of the capital of the pension system in the total capital in the economy in equilibrium according to the "golden rule" (Mac Kellar and Ermolieva, 1999; Myles and Pierson, 2001).

In the case of individuals living in the model for more than two periods, the pension system should provide the optimal choice of consumption for each of the generations, using only two variables of management. The task of central planning, however, remains unchanged – to maximize the discounted amount of benefits for present and future generations. The state pension policy should provide only the optimal choice of consumption for retirees and able-bodied generations living in the same period.

Table 3. Calculation of the Share of the Capital of the Pension System in the Economic Potential of the State when Using the "Golden rule" of the Balance of Pensions.

Calculation Algorithm	Explanation
(1) $R'' = \frac{1+n}{\gamma}$	The steady state of the model, which develops with an optimal trajectory, is characterized by a balanced interest rate close to the sum of the population growth rate and the social discount rate $\frac{1+n}{Y}$
(2) $k^{opt} = \left(\frac{\gamma a A}{1+n}\right)^{\frac{1}{1-a}}$	In the steady state (1) the capital stock in the economy is determined by equation (2). The level of capital intensity (2) corresponds to the modified "golden rule". If the social discount rate is close to zero (social discount factor *Y*=1), the fixed capital stock is determined by the "golden rule".
(3) $k^{GR} = \left(\frac{\gamma a A}{1+n}\right)^{\frac{1}{1-a}}$	The level of capital (3) maximizes per capita consumption. In this case, the usefulness of all future generations in the task of central planning is taken into account with equal weight. In general, when the social discount rate is less than one, the stock of capital in the steady state is less than the stock of capital due to the "golden rule", and the balanced interest rate, on the contrary, is higher.
(4) $PV = -1 + \gamma(1+\beta)\frac{1-\alpha\gamma}{(\gamma+\beta)(1-\alpha)}$	In the steady state, the present value of the pension package as a share of wages is given by equation (4).
(5) $\tau = \frac{\beta}{1+\beta}$	Since the individual savings and the savings of the pension system in the model are interchangeable, any capital structure can exist in equilibrium. For example, if the rate of contributions to the pension system is such that the income after contributions is fully consumed in the first period (in this case, individual savings are zero), all capital in the economy belongs to the pension system.
(6) $s(t) = 1 - \frac{1-\alpha}{\alpha}\left(\frac{\beta}{1+\beta} - \tau\right)$	The share of the capital of the pension system in the total amount of capital in the economy in equilibrium according to the "golden rule" is determined by equation (6).
(7) $s^{MGR}(\tau) = 1 - \frac{1}{\alpha\gamma}\left[(1-\tau)(1-\alpha) - \gamma\frac{1-\gamma\alpha}{\gamma+\beta}\right]$	Similarly, the share of social capital under the modified "golden rule" is described by the following equation. The share of social capital (7) increases with the increase in the contribution to the pension system τ.
(8) $r^{CE} = \frac{\alpha(1+\beta)(1+n)}{\beta(1-\alpha)}$	For different values of the parameters of the production sector and individual preferences, the share of social capital can be an assessment of the extent to which the economy is able to develop on an optimal trajectory without government intervention. Thus, other conditions being equal, an economy with a production function closer to linear requires a larger share of social capital and, consequently, greater state intervention in the pension system 1
(9) $\tau = \frac{\beta}{1+\beta}$	In a stationary state, the economy with a distributive pension system has no social capital. In this case, equality is satisfied.
(10) $(1+n)k^{PAYG} = w(k^{PA\square G}) - \tau - \frac{1}{1+\beta}\left[w(k^{PAYG}) - \tau + \tau\frac{1+n}{R(k^{PAYG})}\right]$	The level of capital in a stationary state can be determined from the following equation, where K_{PAYG} – is the stationary level of capital of an economy with a distributive pension system. The right part of the equation is the total savings of the economy and is a decreasing function of the contribution to the pension system *τ*.
(11) $PV^{PAYG} = /\tau\frac{n-r^{PAYG}}{1+r^{PAYG}}$	Increasing the rate of contributions to the pension system leads to a decrease in individual savings and reduces the stationary level of capital. The present value of the pension package in the steady state as a share of wages is the solution of equation 2.

(12) $k_{t+1} = \frac{\alpha\gamma}{1+n} \times Ak_t^a$	To compare the impact of different pension schemes on the rate of capital accumulation, we write the equation of aggregate savings of capital dynamics. The optimal pension system forces the economy to keep a share $Y\alpha$ of the total output.
(13) $k_{t+1} = \frac{1-\alpha}{(1+n)(1+\beta)} \times Ak_t^a$	An economy without a pension system retains only a share $(1+n)(1+\beta)$ of the total output. Thus, the stock of capital in the competitive equilibrium of the economy without a pension system evolves according to this equation.

Source: Mac Kellar and Ermolieva, 1999; Myles and Pierson, 2001.

Table 4. Formation of the Model of Optimal Distribution of Financial Resources Over Time Between the Generation of Retirees and Able-Bodied Persons.

Calculation Algorithm	Explanation
(1) $(1+n)U'(c_{t-1,j-1}) = \lambda(1+f'(k_t))U'(c_{t,j}),\ j = 2,\ldots,N$ where, $c_{t-1,j-1}$– consumption of the generation that has age j-1, at the time t-1.	First-order conditions are easily modified for the case of many periods. First-best-distribution is characterized by 2(N-1) equations. The first N-1 equations describe the optimal distribution of resources over time.
(2) $\gamma m_{j+1}U'(c_{t,j}) = \beta m_j U'(c_{t,j+1}),\ j = 2,\ldots,N-1$ where, m_j – is the share of the generation that has age j, in the total population.	The conditions for the optimal distribution of resources between generations at any given time are as follows
(3) $U'(c_{t,j}) = \beta(1+r_{j+1})U'(c_{t+1,j+1})j = 2,\ldots,N-1$	Assuming that the structure of the population does not depend on time, equations (1) and (2) give the conditions specified in the equation. The conditions coincide with the condition of the first order of the optimization problem of the individual, which characterizes the optimal distribution of consumption over time

Source: built by the authors according to data.

The formation of a model of optimal distribution of resources over time between the generation of retirees and the able-bodied is shown in Table **4**.

At the stage of numerical modelling, it is assumed that the economy is in a stationary state with a consolidated pension scheme. The government is announcing a policy of transition to a pension system, which consists of a sequence of rates of contributions to the pension system and payments from it. After that, the economy begins to move to a new steady state with a saving scheme in which the stock of social capital is non-zero. The model satisfies the above hypotheses. In this case, there is a problem of equilibrium uncertainty in the multi-period model, as citizens' expectations regarding future interest rates and wages are not defined. The stock of capital k_t in the period of time t depends on the amount of individual savings at the time t, which, in turn, depends on the expectations of citizens regarding the return on capital at times $t+1,\ldots,(t + N - 1)$. Similarly, the decision of individuals to make savings in the future depends on their expectations regarding the level of interest rates and wages in the longer term. It is assumed that the economy after a certain point in time reaches a steady state. The task of numerical modelling is to find a stable trajectory of capital accumulation, which agrees with the potential expectations of the subjects (individuals) of the pension system in a given model.

The stock of capital in the new steady state depends on the parameters of the pension system and can be determined either analytically or by substituting specific values of parameters in the economic and mathematical model of pension provision. This allows to shape the expectations of future retirees in the model and to solve the problem of equilibrium uncertainties. The period of time T, after which the economy reaches a steady state can be and should be much longer than the period of time N after which the transition policy ends.

One of the key points in EMMPP is its sensitivity to the period of time after which the economy is expected to reach a steady state. The duration of reforms is also not strictly defined and depends on resilience to the transition trajectory. In view of this, we will assume that individuals in EMMPP live for 15 periods, which corresponds to the age of 20-65 years. That is, every three years of an individual's life are combined into one period of life in the model. The model assumes that citizens work for the first 11 periods and retire for the last 4 periods.

The initial weights of generations in the total population are determined in accordance with the data on the sex and age structure of Ukraine. After adjusting the model, the ratio of the number of employees to the number of retirees is 1.7:1, which is close to the available estimate. In each period a new generation is born. It is believed that the new generation is $1+n$ (t) times larger than the previous one. Although the rate of population growth in Ukraine today is negative, we will assume that it will increase in time to 0.03, which corresponds to the real rate of population growth of about 1% per year (Aspergen et al., 2019).

Table **5** shows a model for determining the parameters of pension policy, taking into account the fact that a citizen who is j, years old, in the first period of his life chooses the sequence of consumption at each time in order to maximize the discounted utility. It is assumed that the initial value of the coefficient σ does not exceed one, and this corresponds to high elasticity of savings at the interest rate. With the age of the citizen, the coefficient σ increases and his behavior in relation to savings becomes more conservative. In the described model, the initial value of the coefficient σ is equal to 0.6 and with age, the value of the coefficient σ increases to 1.2.

Table 5. Model for Determining the Parameters of State Pension Policy.

Calculation Algorithm	Explanation
(1) $V_t \sum_{j=1}^{N} \prod_{t=1}^{j} \beta_t U(c_{t+j-1,j})$	Formula (1) sets the parameters of pension policy. We will assume that the individual discount factor also changes with the age of citizens. It is worth noting that in the short term the discount rate is much higher than in the long term. Given this hypothesis, the individual discount factor β_j increases over time, i.e. the individual discount rate $\frac{1-\beta_j}{\beta_j}$ decreases with age.
(2) $U = u(c_t) + \beta \sum_{j=1}^{T-t} \delta u(c_{t+1})$	The classic works on hyperbolic discounting provide a form of a utility function. Hyperbolic discounting implies the inability of consumer choice over time. We will assume that the preferences of citizens are capable over time and that the individual discount rate increases with the age of the citizen.
(3) $U_{t,j} = \frac{1}{1-\sigma(o)} \times C_{t,j}^{1-\sigma(j)}$	The utility function is isoelastic, and the relative risk aversion rate changes with the age of citizens
(4) $Y = AKa^{1-a}$	The production sector in the model is defined by the Cobb-Douglas function. It is assumed that the share of capital a in output is equal to 0.6, and the share of labor is 0.4. This choice of parameters is determined by statistical data on the structure of output in Ukraine.

Source: built by the authors according to data.

It is believed that the rate of contributions to the pension system is effective, i.e. takes into account tax evasion. The contribution rate in the model is 20%. The replacement rate in the model is set at 32%. The described initial values of the model parameters determine the steady state of the economy with a consolidated pension scheme.

4. RESULTS

The concept of pension provision in the country is reflected in the characteristic structural features of the mandatory pension system of the modern economy. First, it is the ratio of the saving and distribution schemes, i.e. the share of a person's salary which is paid into the personal savings account and into the State Pension Fund. Second, since employers also contribute to the pension system, the division of responsibilities between the employer and the employee determines the structure of the financial resources of the pension system. The type of pension system depends on the availability of the relevant subjects of the pension system, which are determined by the regulations of the state (Croitoru, 2012; Wong, 2015).

It should be noted that the type of old-age pensions in the pension system covers the largest number of citizens (Fig. **1**), i.e. almost 75% of pensioners receive old-age pensions, the average amount of which in 2019 was 86.83 EUR, in 2020 – 100.48 EUR, in 2021 – 130.49 EUR and almost equal to the average pension for all types of pensions (86.74 EUR in 2019; 101.08 EUR in 2020 and 131.14 EUR in 2021). It is planned that in 2022 the old-age pension will be 2022 148.63 EUR.

For a particular person, the pension system is a mechanism for redistributing money in parts. Therefore, under constant changes in economic conditions, tools are needed to maintain pensions at the level that corresponds to real conditions. One such tool is indexation, which makes it possible to partially or fully recoup the rise in prices for consumer goods and services. In world practice, pension funds play an important role in ensuring the investment process, which through the banking system capitalizes a large share of their assets in GDP (Lybanova, 2014; Pikus and Khemii, 2017). The money accumulated in pension funds is an important amount of investment resources in the direction of forming interconnected activities of the pension system of Ukraine with banks. Cash flows of pension funds sent through banking institutions for the payment of pensions in 2016 amounted to 5.04 billion EUR (or 57.8% of the total amount of pension recipients), (Mischuk and Shushkina, 2016). In 2016, the Pension Fund together with Oschadbank introduced the issuance of electronic pension certificates. They are also bankcards and were issued to pensioners – IDPs (Distribution of banks…, 2019). In 2017, pensions 58% of recipients (6.9 million people) were received through banking institutions, in 2018 – 60% (7 million people), в 2019 – 62.7% (7.2 million people), в 2020 – 66.1% (7.5 million people), в 2021 – 70.4 (7.8 million people) (Diamond, 2011; Croitoru, 2015; Aspergen et al., 2019).

The interaction of the Pension Fund with banking institutions has allowed the state to save the redistribution of financial resources in the expenditure part of the budget by more than 10.2 million EUR in 2020 and 2.19 million EUR in 2021. At the same time, such cooperation encourages banks to expand the range of customers in electronic payment systems, increase cash flows for pensions, as bank non-cash payments to customers. The volume of pension payments through electronic payment systems in banking institutions is constantly increasing: in 2018 – 5.99 billion EUR (63.8% of expenditure on pension payments), which is 832.79 million EUR (16.2%) more billion EUR (68.3% of pension expenditures), which is 1.90 billion EUR more than in 2018; in 2020 – 94.36 billion EUR (69.7% expenditure on pension benefits), which is 1.54 billion EUR more than in 2019; in 2021 –

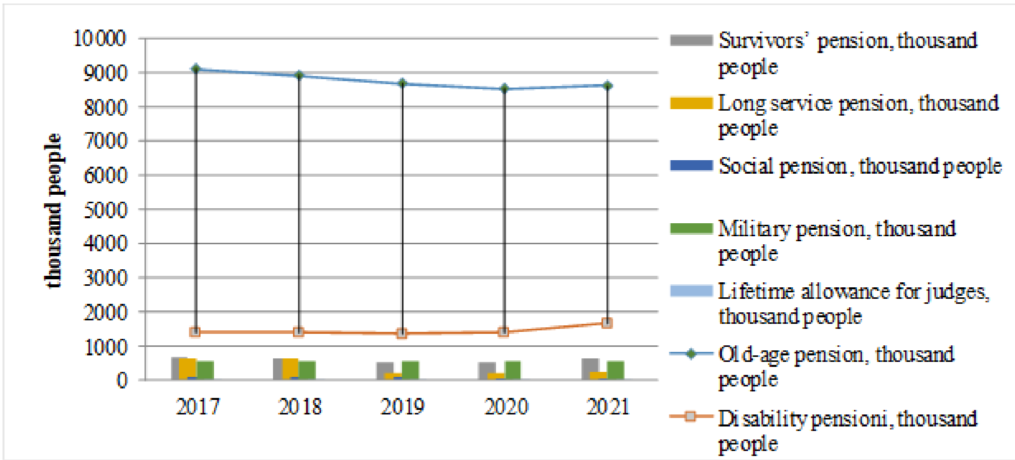

Fig. (1). Distribution of the number of pensioners by types of pensions.
Source: built by the authors according to data.

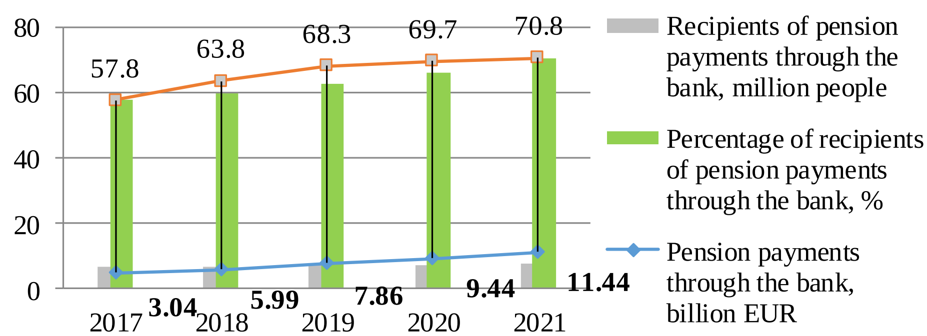

Fig. (2). Payment of pensions through banking institutions of Ukraine for 2017-2021.
Source: built by the authors according to data.

11.34 billion EUR (70.8% expenditures on pension benefits), which is 1.9 billion EUR more than in 2020 (Fig. **2**).

One of the main activities of the bank in the pension system is to perform the functions of the custodian bank of the private pension fund. It opens and maintains pension fund accounts; securities transactions, verification of the calculation of the net asset value of the pension fund and the net value of a unit of pension contributions; execution of the administrator's orders in accordance with the law; execution of orders of the person managing the assets of the pension fund, according to the investment declaration of the pension fund and others (Blanchet and Kessler, 1991). The Bank has property liability for losses caused to the pension fund due to non-fulfilment of obligations regarding the activities of the Banking Management Fund (BMF) and provides management of its funds to the participant for the formation of the investment portfolio; asset valuation; determining the value of net assets; ensuring the preservation of assets; formation of reserves, distribution of income; payment of additional pension and target payments; transfer of the participant's funds to another one etc. according to the legislation (Holzmann, 2013).

The high level of protection of the population under the pension system in Ukraine and European countries is demonstrated by the coverage ratio of pension benefits (Fig. **3**). We note that most countries build their pension system on the basis of the substitution rate because the subsistence level can be significantly lower in relation to current earnings (Diamond, 2011; Wong, 2015).

In Ukraine, the substitution coefficient in terms of types of pensions during 2017-2021 exceeded its minimum level by 40% (Fig. **4**). However, during this period there was a steady downward trend in this indicator. Thus, in 2021 the substitution ratio decreased by 18.4% compared to 2019. This indicates a significant impact of external factors on determining the parameters of pension reform and the lack of a systematic approach to regulating pension benefits in the state pension insurance.

The pension substitution ratio in Great Britain is 29%, the United States – 37%, Belgium – 39%, Germany – 41%, the replacement rate is at the same level as in Ukraine (34%). These countries, in contrast to Ukraine, are characterized by a well-developed system of voluntary pension insurance,

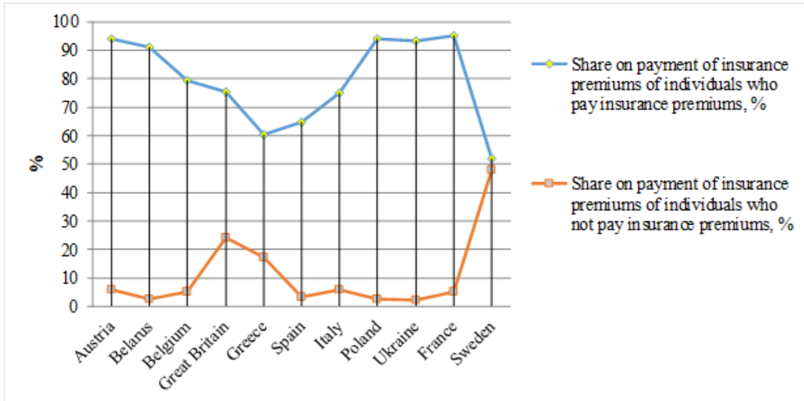

Fig. (3). Coverage ratio of old-age pensions in Ukraine and in the EU countries for 2021, %
Source: built by the authors according to data.

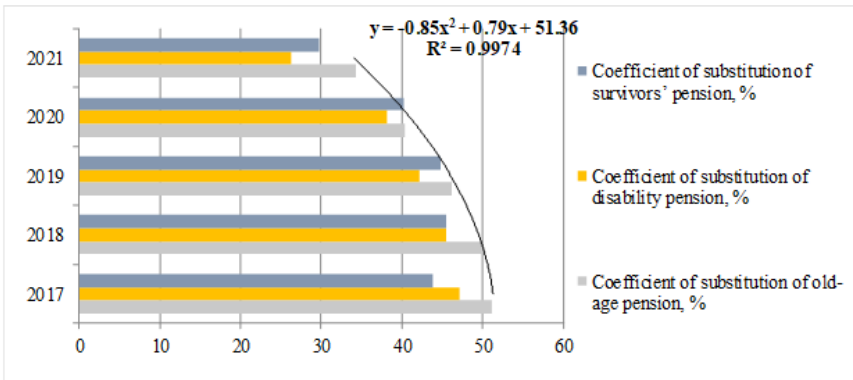

Fig. (4). Coefficient of substitution of pensions by age, disability and in case of loss of breadwinner in Ukraine for 2017-2021, %
Source: built by the authors according to data.

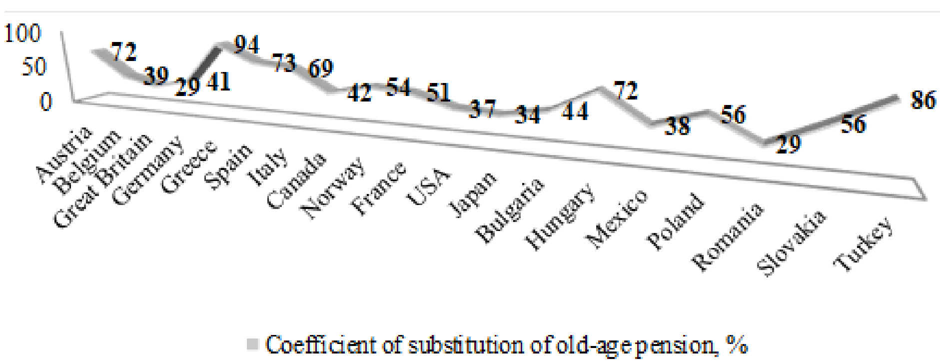

Fig. (5). Coefficient of substitution of old-age pensions in the countries of the world for 2021,%
Source: built by the authors according to data.

which is able to ensure a high overall level of pension benefits (Fig. **5**).

The demographic crisis, changes in the social structure of the population, fluctuations in the number of employees, disparities between the payers of the Single Social Contribution (SSC) and retirees have significantly affected the overall pension system in Ukraine due to reduced employment. In 2017-2021, the ratio of the number of employees to the number of pensioners is gradually declining, and in 2021, it was 1.34 (i.e., one employee maintains one pensioner). This indicates the inefficiency of the consolidated pension scheme because, for the effectiveness of this process, one pensioner must have five contributors (Fig. **6**).

It should be noted that the state of the consolidated pension scheme, first of all, depends on how widely it covers the working population by the insurance premium. As in Ukraine, this ratio is one of the lowest in Europe, according to the World Bank, the load factor of the pension system of Ukraine over the next 20-25 years may decrease to 0.67 (Fig. **7**), i.e. the number of retirees will be greater than the number of employees per third.

Fig. (6). The ratio of the number of employees to the number of retirees in Ukraine for 2017-2021, million people.
Source: built by the authors according to data.

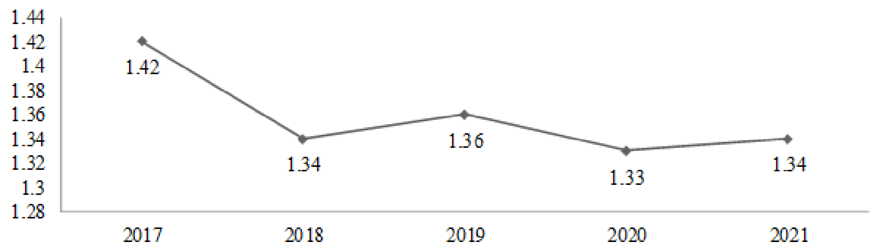

Fig. (7). The load factor of the pension system in Ukraine for 2017-2021.
Source: built by the authors according to data.

The low workload rate of the Ukrainian pension system is due to the high level of wage shadowing in Ukraine. This trend may lead to an unbalanced demographic burden on the working population, as well as a further increase in the budget deficit of the Pension Fund of Ukraine. Therefore, in the time horizon, the "golden rule" of the equilibrium of the pension system of Ukraine should be achieved with an alternative state of the social contribution rate at the level of 5%.

This will minimize the negative impact of the single social contribution (SSC) and ensure the introduction of a saving pension scheme in the stationary state of the normalized replacement rate of 50%. In this case, the share of the capital of the pension system in the total economy of the state should not exceed 14%.

5. DISCUSSION

According to the economic-mathematical model of pension provision, the length of the time horizon of the pension provision of the future pensioner consists of 3 years $(\Delta = \frac{65-20}{15} = 3 years)$. That is, the time horizon of future retirees from 20 to 65 years, formalizes the optimization model of pension and social security through the introduction of the saving system as a tool of domestic long-term investment resources to attract and accumulate funds in private pension funds during employment.

Activation of the accumulative principles of pension contributions with the subsequent introduction of elements of investment, on the basis of information technology of the pension system allows modelling the optimization processes of effective social protection. It is established that the fastest rate of population decline will be due to individuals of working age. If the current birth, death and migration rates are maintained, the population is projected to decrease by 2050 by 28%, while the share of people aged 60 and over will increase to 32%. The average version of the forecast provides a gradual increase in the total fertility rate to 1.8 children per 1 woman in 2050. This will increase life expectancy by an average of 5 years. It should be noted that under such conditions, the population will decrease by 18% and the level of ageing will increase to 34%. The ratio between the population of retirement and working age will almost double. The demographic changes that will be observed in Ukraine will be in line with global trends and will indicate a very rapid pace of development of the consolidated pension scheme (Kornieiev, 2013) The introduction of consolidated pension provision will increase pension benefits through mandatory savings of system participants as financial resources for investing in a three-level state pension insurance system, in which consolidated scheme is complemented by two levels of saving scheme: the second - mandatory and the third – voluntary. Investment of pension assets and their protection with the help of financial institutions in the securities market through the placement, preservation and management of financial resources of state and non-state pension funds are due to probabilistic signs of risks that may affect the achievement of goals. Investing pension assets in securities of international exchanges will significantly increase guarantees of accumulation of future payments on pension accounts and create a reserve protection function of the state against inflation.

The market of insurance services is the second in terms of capitalization of financial resources in the system of saving pension provision. Thus, the share of insurance premiums received from individuals for 9 months of 2020 was 49.6%, from legal entities – 40.9 %, from reinsurers – 9.5%. Insurance premiums in the amount of 0.21 billion EUR or 19.5% were transferred to reinsurance of saving pension funds, of which 0.09 billion EUR were transferred to non-resident reinsurers (Lybanova, 2014; Martynenko, 2017b; Ponomarenko et al., 2017). The total amount of expenditures on pensions and other planned payments in 2021 amounted to 6.92 billion EUR against 6.45 billion EUR in 2020. At present, in order to optimize financial resources and balance the budget of the Pension Fund, it is necessary to ensure financial stability and solvency with the intensification of information technology to monitor the activities of the financial services market in order to forecast the mandatory saving pension provision.

It should be noted that the introduction of the saving component of the pension system of Ukraine requires additional expenditures from the State budget, which can be covered by long-term investment resources – a saving asset of potential entities (individuals) of the three-level pension system, and social guarantees (Table **5**). Thus, the projected value of indicators in the implementation of the saving component of the pension system indicates self-sufficiency. Thus, according to forecasts, there will be a decrease in the number of payers of pension contributions (potential entities (individuals) of the three-level pension system and social guarantees) based on the demographic situation in the country. At the same time, the saving assets of potential entities (individuals) will increase at the expense of participants in pension contributions under the age of 35, as tax law regulates the payment of compulsory insurance contributions. Thus, the definition of opportunities to improve the conditions of redistribution of savings (pension contributions) of potential entities (individuals) of the pension system remains relevant.

Table 5. Indicators of the Saving Component of the Pension System of Ukraine.

Indicators	2022	2023	2024	2025
Net cash flow of the Accumulation Fund, million EUR	1.87	2.49	10.58	20.71
Net income from the activities of the Accumulation Fund, million EUR	2.47	3.81	7.44	11.18
The number of contributors, thousand people	11.74	11.07	11.00	10.91
Insurance premiums, million EUR	2.14	2.59	3.44	4.19
Accumulation assets of employees, million EUR	3.48	7.14	11.47	15.57
Total investments of large and medium-sized businesses to bring activities to compliance with the requirements of the authorization of the second level, thousands EUR	174.42	274.63	288.12	x
The total average annual costs of large and medium-sized businesses to bring activities in line with the requirements of the authorization of the second level, million EUR	1.88	2.35	2.78	x

Source: calculated by the authors

In the interaction of elements of the risk-oriented saving pension scheme, we considered the possibility of investing pension savings as a long-term investment resource in the economy, given the need to create conditions for an optimal long-term investment plan at all levels of the pension system; establishment of conditions for placement of saving assets of potential subjects (individuals) of the pension system in a three-level cluster of pension provision.

The establishment of the optimal plan for the saving of pension contributions between the levels of the pension system with minimal investment was carried out with the solution of the transport problem of linear programming (Filonenko, 2014; Mischuk and Shyshkina, 2016). It is noted that (m-3) points of the pension system levels (A_1, A_2, A_3), in which saving pension contributions are placed (a_1, a_2, a_3).These pension contributions are interconnected in a system of n points (B_1, B_2, B_3) with the volume of demand (b_1, b_2, b_3).It is possible to transport from each supply point to each consumption point, i.e. from the depositor to the levels of the pension system. At the same time, the condition is met that the total amount of saving assets of the depositor is equal to the total demand for pension contributions at the levels of the pension system. The value C_{ij} of pension savings from each A_i-го depositor to B_j-го levels of the pension system is equal to (i = 1, m; j = 1)). Using the requirements for setting the transport problem of linear programming (Filonenko, 2014; Mischuk and Shyshkina, 2016), a matrix of the value of pension savings is proposed, which has the form:

$$C_{ij} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{pmatrix}.$$

To achieve this goal, a plan is defined in which all pension savings in the amount of 15.57 million EUR are distributed among the three-level cluster of the pension system to ensure the financial interests of depositors, i.e., the subjects of saving pension provision (Filonenko, 2014; Mischuk and Shyshkina, 2016):

$$\sum_{i=1}^{m} a = \sum_{j=1}^{n} b \tag{11}$$

The mathematical formulation of the problem must correspond to the following values of the objective function (Filonenko, 2014; Mischuk and Shyshkina, 2016):

$$Z = \sum_{i=1}^{m\Sigma} \sum_{j=1}^{n\Sigma} C_{ij} \times X_{ij} \to min \tag{12}$$

With restrictions:

by the amount of saved assets of employees (Filonenko, 2014; Mischuk and Shyshkina, 2016):

$$\sum_{j}^{n} x_{ij} = a_i ; i = 1, m \tag{13}$$

by volume of placement to the levels of the pension system (Filonenko, 2014; Mischuk and Shyshkina, 2016):

$$\sum_{j}^{n} x_{ij} = b_j ; j = 1, n \tag{14}$$

non-negative variables (Filonenko, 2014; Mischuk and Shyshkina, 2016):

Table 6. The Optimal Amount of Placement of Saved Assets of Potential Entities (Individuals) in the Three-Level Cluster of the Pension System of Ukraine.

Subjects of the Pension System	Variability of Investment			The Presence of Accumulated Active Subjects of the Pension System
	First Level	Second Level	Third Level	
First level, million EUR	1 –	3 4.20	2 –	4.20
Second level, million EUR	2 –	1 1.87	3 5.29	7.16
Third level, million EUR	3 3.74	2 0.47	1 –	4.21
The need for the accumulation of assets of the subjects of the pension system, million EUR	3.74	6.54	5.29	15.57

Source: calculated by the authors.

$$x_{ij} \geq 0; i = 1, m; j = 1, n \quad (15)$$

The model of distribution of saved assets of potential subjects (individuals) of the pension system among all participants has the form:

Availability of saved assets of potential subjects (individuals) of the pension system, million EUR:

$$x_{11}+x_{12}+x_{13} \leq 4.20;$$
$$x_{21}+x_{22}+x_{23} \leq 7.17;$$
$$x_{31}+x_{32}+x_{33} \leq 4.20 \quad (16)$$

Need to save assets of potential subjects (individuals) of the pension system, million EUR:

$$x_{11}+x_{12}+x_{13} \leq 3.74;$$
$$x_{21}+x_{22}+x_{23} \leq 6.54;$$
$$x_{31}+x_{32}+x_{33} \leq 5.29 \quad (17)$$

In order to compile a dual problem, we replace the variables x_{ij} under condition (13) with u_1, u_2, u_i,…, u_m, and the variables x_{ij} under condition (14) with v_1, v_2, v_j,…, v_n. The objective function will look like: $F = 3.74u_1 + 6.54u_2 + 5.29u_3 + 4.20v_1 + 7.17v_2 + 4.20v_3 \rightarrow min$

The optimality of the reference plan is checked. Preliminary potentials u_i, v_j on the occupied cells of Table **6** are found, in which $u_i + v_j = c_{ij}$, and, assuming that $u_i = 0$. Then,

$$u_1 + v_2 = 3; 0 + v_2 = 3; v_2 = 3$$
$$u_2 + v_2 = 1; 3 + u_2 = 1; u_2 = -2$$
$$u_2 + v_3 = 3; -2 + v_3 = 3; v_2 = 5$$
$$u_3 + v_2 = 2; -3 + u_3 = 2; u_3 = -1$$
$$u_3 + v_1 = 3; -1 + v_1 = 3; v_1 = 4 \quad (18)$$

The reference plan is optimal, as all estimates of free cells satisfy the condition $u_i + v_j \leq c_{ij}$ (Table **6**).

Thus, the proposed saving of pension contributions proves a balance between the availability and need for pension savings between potential entities (individuals) of the pension system. It is established that with this optimal plan in all these three options, the total income from pension savings will be:

$$F = 3\times 4.20 + 1\times 1.87 + 3\times 5.29 + 3\times 3.74 + 2\times 0.47 = 42.5 \; million\,EUR \rightarrow min \quad (19)$$

The results of the redistribution of financial resources as potential pension investments in the three-level cluster of the pension system confirmed the close relationship between household incomes from long-term pension savings. It is the optimal plan for the redistribution of pension assets that will ensure the diversification of financial resources, which will help minimize risks in the implementation of the mechanism of the saving pension scheme.

It is established that depending on the share of pension savings, the total income from pension savings will be 42.50 million EUR. This will create conditions for proper protection of pension system entities from the impact of external and internal risks. A positive factor of the proposed optimization plan for the redistribution of pension assets is the allocation of financial resources to support the mandatory pension provision.

6. CONCLUSIONS

Thus, the optimal structure of financing three-level pension schemes based on transfers in a simple deterministic model with the same number of potential subjects (individuals) of the pension system generates profits and wages as endogenous factors that depend on the rate of savings. This allows optimizing the financing policy of the pension system in a general equilibrium, where funding affects return on investment and wages through the impact on capital formation in the context of irregular demographic evolution, taking into account the criteria of intergenerational welfare used to develop optimal pension assets. It should be noted that the generation of working citizens in the saving system saves their own financial resources just as much as in the consolidated pension system. However, the savings of the older generation are declining. This phenomenon may be due to the substitution effect, which equates to an increase in savings and is caused by the rising incomes of citizens. In addition, under the new pension system, retirees will be able to receive higher pension benefits and will not need significant savings of their own until retirement.

Changing the pension legislation and its perception by society requires a special pension philosophy, namely the awareness that a person at the beginning of the work should think about financial security for himself in old age. At the same time, the state, in turn, must ensure decent working conditions and retirement from its own savings and the right to state protection. Regulation and supervision of insurance companies ensure Ukraine's saving pension scheme that meets the requirements for a sufficient state solvency margin for future pension payments. Pension Fund of Ukraine provided payment of pensions and cash benefits in full in all regions. The combination of distribution and saving pension schemes will allow investing pension funds, determining the criteria for optimizing the investment process in developing a mechanism for allocating financial resources of the pension system of Ukraine as pension savings.

The construction of the pension provision mechanism that regulates the saving pension system reflects a practical model of an effective automated system for potential subjects (individuals) of the saving component. The implementation of this mechanism is based on the optimal redistribution of saved assets in the three-level pension cluster using information technology, reduces the risk of investing pension assets in the economy and increases the confidence of investment participants in effective management and proper state regulation of pensions.

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