The Contribution of Spatial Econometrics in the Field of Empirical Finance

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Abstract: Spatial econometrics is a subset of econometric methods evolved from the need to account for the location and spatial interaction. This means that what happens in one economic unit of analysis is not independent of what happens in neighboring economic units. Spatial econometric methods have been advanced quickly and many studies show the usefulness of these techniques in various fields. However, they have not yet received sufficient attention in empirical finance. So, this article asks the question: what should a financier who wishes to use regression models involving spatial data know about spatial econometric methods? More precisely, this paper has two goals. In the one hand, it attempts to present a review of the peculiarities of spatial econometrics, and, in the other hand, it discusses the application of spatial econometrics in the field of finance. It summarizes some of the different spatial econometrics models that have been used in finance, and describes different kind of economic and financial distance.

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JEL Classification: C31, C21, G10, G20.

INTRODUCTION

Units of analysis are most of the time treated as isolate entities and the regression models have assumed that observations are independent of one another. However, evidence in favor of spatial interactions is now well justified in the empirical literature. LeSage (2008) notices that "It is commonly observed that sample data collected for regions or points in space are not independent, but rather spatially dependent, which means that observations from one location tend to exhibit values similar to those from nearby locations".

The specific characteristics of spatial data break down the basic assumptions made by the standard econometric tools, that each observation is independent of other observations. In this regard, spatial econometrics, a subfield of econometrics, has recently emerged as a useful tool to study spatial interaction effects among geographical units (neighborhoods, municipalities, counties, regions, states, countries, etc.) that are in close geographical proximity.

Even though spatial econometrics has been advanced quickly and it has been a rapidly evolving discipline, it has not yet received much attention in finance, as mentioned by Fernandez (2011) "It has essentially been overlooked in other subfields of economics and in the field of finance as a whole". Thus, the main objective of this paper is to review what should a financier, wishing to use spatial regression methods in applied work, know about spatial econometrics. The reader can find various books on spatial econometrics, e.g., Anselin (1988); Arbia (2006); LeSage and Pace (2009); Elhorst (2014); among others, which by their nature and length can furnish a more detailed view and scope of this particular subject. In this papers, a brief survey of the basic tools and models from spatial econometrics will be provided.

This paper contributes to two strands of literature. First, it provides a comprehensive review of the subject of spatial econometrics and describes the evolution of this field and its characteristics. Second, it discusses the application of spatial econometrics in the field of finance: it summarizes some of the applied studies in spatial econometrics in the field of finance and it describes various kinds of financial distance.

The remainder of this analysis is organized as follows. After the introduction, Section 1 reviews the literature on spatial econometrics. Section 2 treats the specification of spatial regression models. Section 3 describes some application of spatial econometrics in the field of finance. Finally, the conclusion is presented.

I. SPATIAL ECONOMETRICS

1. Development of spatial econometrics

1.1. Definitions

Spatial econometrics is a sub-field of econometrics that evolves from the need to account for spatial aspects. As mentioned by Anselin (2010), there are three main definitions of

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spatial econometrics in the literature formulated at different points in time over the last thirty years.

To start, the term spatial econometrics was originated by the Belgian economist Jean Paelinck in the early 1970s[1]. It designates a field of applied econometrics that encompasses methodological aspects to deal with estimation and specification problems of spatial autocorrelation, in the application of regional and multiregional econometric models (Anselin and Rey 1997).

In their classic book "Spatial Econometrics", Paelinck and Klaassen (1979) did not give a clear definition of spatial econometrics, however they started by outlining five fundamental characteristics of this field:

(1) the role of spatial interdependence; (2) the asymmetry of spatial relations; (3) the importance of explanatory factors located in other spaces; (4) the differentiation between ex-post and ex-ante interaction and (5) the explicit modelling of space.

Almost twenty years later, the definition of spatial econometrics has broadened from the cross-sectional setting to the space-time domain. The subject of spatial econometrics is outlined as "a subset of econometric methods that is concerned with spatial aspects present in cross-sectional and space-time observations" (Anselin 2006).

1.2. Historical Overview

In recent years, models that explicitly incorporate space has become very popular in the economics literature. Spatial econometrics is a promptly evolving discipline. It has been advanced quickly thanks to the effort of a large number of spatial econometricians such as Anselin, Arbia, Baltagi, Bera, Elhorst, Fingleton, Florax, Getis, Kapoor, Vega, Kelejian, Lacombe, Lee, LeSage, Paelinck, Pfaffermayr, Piras, Prucha, Rey, Yu, Le Gallo, etc.

In his article "Thirty years of spatial econometrics", Anselin (2010) presents an overview of the development of spatial econometrics in the past three decades. He distinguishes three broad phases of development of this field: The first stage labeled preconditions for growth (from the early 1970s to the late 1980s), the second stage is the take-off (dated in the 1990s) and the final stage is the maturity (attained in the early twenty-first century).

While, Arbia (2011) in his article "Lustrum of SEA: Recent Research Trends Following the Creation of the Spatial Econometrics Association (2007_2011)" tries to identify recent trends and hot topics of spatial econometrics by exploring 237 papers devoted to the subject that were published from 2007 to 2011 in various scientific journals.

Originally, spatial econometrics appeared principally in specialized fields such as regional science and geographical analysis (early reviews can be found in Paelinck and Klaassen (1979); Cliff and Ord (1973,1981); Hordijk (1979); Upton and Fingleton (1985); Anselin (1988, 1992); Hainingb (1990); Anselin and Florax (1995); Anselin and Aera (1997); Cressie (1993)). However, the situation has changed dramatically. More recently, the methodology of spatial econometrics has seen a virtual explosion. It has extensively been studied in a wide range of empirical investigations in more traditional fields of economics including, among others, international economics, labor economics, public economics, agricultural economics, environmental economics, resources and energy economics, international trade, real estate analyses, political sciences, innovation diffusion.

1.3. Reasons of the Development of Spatial Econometrics

Anselin (2010) supports that the attention to spatial data analysis is no longer obscure and is moved from the margins (urban and regional economic analysis) to the mainstream (economics and other social sciences), he adds that there is a potential explosion in the number of articles and textbooks dealing with spatial econometrics[2].

The inclusion of spatial effects in regression models is typically motivated either on theoretical grounds as well as on a practical ground. The development of new theoretical economics such as the geographical economics, the industrial economics and the international economics have generalized the consideration of spatial interactions in the analysis of agent's economic decisions (Le Gallo 2002). These new theoretical frameworks of "social interaction" and "interacting agents" explain how individuals belonging to the same group have the same behavior. According to Manski (1993), Anselin and Bera (1998), Le Gallo (2002) and Anselin (2001, 2002), these effects receive different designations in various subfields, such as social norms, neighborhood effects, social interactions, peer group effects, strategic interaction, copycatting, conformity, contagion, yardstick competition, to name a few.

The second motivation behind the increased interest in spatial techniques is the growing availability of spatial data and their peculiarities. As documented by Anselin (2001), the need to handle spatial data "has been stimulated by the explosive diffusion of geographic information systems (GIS) and the associated availability of geocoded data". Additionally, statistical and econometric tools for detecting spatial effects as well as econometric models have been developed in the literature since the late 1980s. For example, LeSage and Pace (2009) provide *Matlab programs*, Anselin develops the project "*GeoDa*" which is a free software for spatial analysis, and there is also, the development of the software *R*.

1.4. Spatial Econometrics VS Standard Econometrics

To better understand spatial econometrics, it is often useful to make a comparison with standard econometrics, namely time series econometrics or non-spatial econometrics.

According to Varga (1998), in time series, observation at a point in time is followed by the value of the same variable detected at the next time point and that time series observations generally occur at equally spaced time points. In stand-

[1] Even though 1979 is a convenient historical starting point for spatial econometrics, it should be indicated that the introduction of the term spatial econometrics could be dated back to Cliff and Ord (1973).

[2] Arbia (2011) counted 237 papers dealing with spatial econometrics that appeared in various scientific journals from 2007 to 2011.

ards econometrics, temporal autocorrelation is unidirectional (dependence in time dimension only), where only the past influences the future.

In spatial econometrics, spatial autocorrelation is multidirectional (dependence both in time and space dimensions) and observations are dependent, which leads to a spillover of information from one place to another, there's a feedback and simultaneity. This bouncing around in space is the major difference between time series analysis (where there's no coming back in space) and spatial analysis. Elhorst (2014) stresses that spatial econometrics is not an extension of time-series econometrics to two dimensions. The main difference is that in spatial econometrics two geographical units can impact each other mutually.

2. Spatial Effects

In contrast to standard econometrics, spatial econometrics methods aim at taking into consideration spatial effects. These effects pertain to two kinds of specifications: spatial dependence (spatial interaction) and spatial heterogeneity (spatial structure). Anselin (1988) considers these spatial effects as the essential reason for the existence of a separate field of spatial econometrics.

2.1. Spatial Dependence

According to Anselin and Bera (1998), spatial dependence or spatial autocorrelation[3] refers to the coincidence of value similarity and locational similarity. Anselin (1988) defines spatial dependence as a functional relationship between what happens at one geographical unit and what happens elsewhere. Varga (1998) supposes that spatial dependence is a rule than an exception. It is a phenomenon commonly encountered in spatial analysis and it's related to The First Law of Geography (Tobler 1979) in which "everything is related to everything else, but near things are more related than distant things".

Otherwise, spatial dependence is a situation where values observed at one location i depend on the values of neighboring observations at nearby locations j (LeSage and Peace 2009). More formally, the existence of spatial dependence may be expressed as follows:

$$y_i = f(y_j) \qquad for\ i \neq j \quad (1)$$

Or

$$Cov\ (y_i, y_j) = E(y_i y_j) - E(y_i)\ .\ E(y_j) \neq 0 \qquad for\ i \neq j \quad (2)$$

Where y_i and y_j are the values of the focus variable respectively at locations i and j in space.

Spatial dependence can be either positive or negative. Positive spatial autocorrelation occurs when similar values (high or low values) of the studied variable tend to cluster in space (similarity or clustering). Negative spatial autocorrelation means that each location tends to be surrounded by neighboring locations with very dissimilar values. The absence of spatial autocorrelation indicates randomness and that the location of data is not related in space.

Anselin (1988a) argues that dependence in space may be caused by a variety of measurement problems and complex patterns of dependencies and spatial interactions. Also, Varga (1998) states that spatial dependence may be the result of two separate phenomena, one technical and the other fundamental. Anselin (1988) presents some examples of measurement error such as the arbitrary delineation of spatial units of observation (e.g. county boundaries), problems of spatial aggregation[4], spatial externalities and spill-over effects.

The second factor of spatial dependence is more fundamental. Dependence among what happens at one point in space and what happens elsewhere can be present even the aggregation of data is perfect. It's linked to the existence of a variety of spatial interaction phenomena and the importance of space in explaining human behavior. Spatial autocorrelation comes from the fact that the intensity of interaction is determined by distance. As a consequence, what is happens at one point is determined (in part) by what observed elsewhere in the system. And it must be noted that the size of interactions is negatively related to distance. This means that as data locations become more dispersed, dependence becomes weaker (Cressie 1993; Varga 1998).

The existence of spatial autocorrelation can be identified in a number of ways. The most used test is the Moran's I statistics (Cliff and Ord 1981)[5], and it is formally presented as follows:

$$I = \frac{n}{\sum_i \sum_j W_{ij}} \frac{\sum_i \sum_j W_{ij}(y_i - \bar{y})(y_j - \bar{y})}{\sum_i \ (y_i - \bar{y})^2} \quad (3)$$

Where n is the number of observations, y_i and y_j are the variable values at location i and location j respectively, $i \# j$. W_{ij} are elements of the spatial weights matrix measuring the spatial proximity between location i and j (see paragraph 3). Moran's I varies between -1 and $+1$. A Moran's I close to zero denotes the absence of spatial correlation. A significantly positive value of Moran's I indicates positive autocorrelation while, a significantly negative value indicates negative autocorrelation.

2.2. Spatial Heterogeneity

The second class of spatial effect is spatial heterogeneity. According to Anselin *et al.*, (2008) and Anselin (2010), spatial heterogeneity is a special case of a familiar problem in standard econometrics: heterogeneity.

Spatial heterogeneity is a consequence of the lack of stability (uniformity) of the behavioral or other relationships over

[3] As Anselin and bera (1998), we will use the terms of spatial dependence and spatial autocorrelation interchangeably.

[4] Data used for analysis is collected only at an aggregate scale. Therefore, Anselin (1988, p.12) noticed that "there may be little correspondence between the spatial scope of the phenomenon under study and the delineation of the spatial units of observation".

[5] Others tests can be found in the literature such as the Moran's scatter plot, Geary's C statistic and The local Moran's I_i test.

space (the spatial units of observation are not homogeneous; they are distributed unevenly in the area) (Anselin 1988). For example, several factors, such as census tracts (with different area and shape), country or regions that vary by population, by incomes levels, by industrial production or by their various degrees of technological development, the existence of leading and lagging regions, etc., would argue for taking into account the particular features of each location or spatial unit. Spatial heterogeneity can be materialized by spatially varying coefficients (structural instability) or by varying error variances through observations (groupwise heteroscedasticity[6]) (Dall'erba and Le Gallo 2008; Anselin 1988, 2001, 2010; Le Gallo et al. 2006).

Spatial autocorrelation and spatial heterogeneity often appear together in spatial econometrics studies but they are not identical concepts. The relation between these effects is complex and the distinction between them is not always obvious (Anselin 1988, 2010; Anselin and Bera 1998; Le Gallo et al. 2006).

The presence of spatial dependence has the consequence of making standard econometric techniques not applicable and results obtained from these techniques are not valid. In this regard, LeSage (2014) argues that the fundamental assumption made by the ordinary regression methods, namely, that each observation is independent of other observations is typically violated by spatial data. As a consequence, the conventional Ordinary Least Squares (OLS) estimators may turn out inefficient or inconsistent (Anselin 1988, 2010; LeSage 1999 and Ertur and Kalidou 2005) and, a spatial econometric method seems to be an adequate alternative.

3. Spatial Weights Matrix and Spatial Lag Operator

3.1. Spatial Weights Matrix

When proving appropriate representation of dependence, the spatial arrangement of observation should be considered. To model spatial heterogeneity, the spatial position of observations has to be accounted for (Varga 1998). Formally, this refers to the topological notion of "proximity" and "neighborhood", where observations are ordered into neighborhood classes according to their relative distance.

The spatial weights matrix W is used to capture proximity and relative position of geographic locations in space. It measures how an observation at one location depends on other observations at other neighboring locations and, each weight represents different dependencies among these spatial units.

The matrix W, exogenously defined, is an N by N positive and symmetric matrix, with weight elements W_{ij} expressing interaction or dependence between spatial units. If the observations i and j are connected, then W_{ij}, has a value greater than 0. However by convention all the diagonal elements of the weights matrix equal to zero ($W_{ii} = 0$) (an observation is not a neighbor to itself). For computational simplicity and for easier interpretation of the spatial variables, spatial weight matrices are usually row-standardized, thus, every element in the *ith* row is divided by the row sum, it can be written as follows:

$$W_{ij}{}^{S} = \quad W_{ij}/\textstyle\sum_j W_{ij} \qquad ,\forall\, i \quad (4)$$

The elements in each row in W^S sum to one $(\sum_{j\neq i}^{N} W_{ij}{}^{S} = 1,\ for\ i = 1, \ldots, N)$ and each weights $W_{ij}{}^{S}$ take values between 0 and 1. Therefore the weights no longer express absolute values but relative ones. As a consequence, comparison of parameters estimates resulting from different spatial models became more reasonable.

To determine spatial connectivity, two main conceptions based on geographical information are used: binary contiguity matrix and distance weighting matrix (Anselin 1988, 2002; Anselin and Bera 1998; Ertur and Kalidou 2005; Le Gallo 2000).

The original measures for spatial dependence are advanced by Moran (1948) and Geary (1954) and there are based on the concept of binary contiguity between spatial units (being or not being a neighbor) (Anselin 1988; Ertur and Kalidou 2005). The spatial weights are supposed to be 1 if two spatial neighborhoods share a common border and 0 otherwise. The elements of the connectivity matrix $W = (W_{ij})$ are defined as follows:

$$W_{ij} = \begin{cases} 1 & (if\ i,j\ share\ a\ common\ border) \\ 0 & (otherwise) \end{cases} \quad (5)$$

According to Anselin (2002), there are three form of Weight matrices based on the binary contiguity: the rook contiguity (the weights equal 1 if the two regions share common boundaries and 0 otherwise), the bishop contiguity (the weights equal 1 if the two regions share common vertices and 0 otherwise) and the queen contiguity (the weights equal 1 if the two regions share both boundaries and vertices and 0 otherwise). However, the binary contiguity has some shortcomings, it does not allow to represent well the strong dependence relationships between spatial units.

By extending the simple concept of binary contiguity, Cliff and Ord (1973, 1981) suggested a new spatial weight matrix W, also known as "Cliff-Ord weight matrix", to include geographic distances between spatial units. The original suggestion by Cliff and Ord consists of combining the length of the common border between two spatial units and the distance measures (inverse distance, or negative exponentials of distance). In that sense, Anselin (2002) considers that two units i and j are neighbors if the distance between them is less than a given critical value. Thus, Cliff-Ord weights may be expressed as:

$$W_{ij} = \begin{cases} (d_{ij})^{-a}(\beta_{ij})^{b}; & \forall\, i \neq j \\ W_{ii} = 0 & \end{cases} \quad (6)$$

Where a and b are fixed parameters, d_{ij} is the distance between two spatial units i and j and, β_{ij} the proportion of the common border between units i and j in the perimeter of i.

[6] If heterogeneity is reflected in measurement errors (missing variables, functional misspecification) it may result in heteroscedasticity (Anselin, 1988, p.13)

Yet, empirical studies imply much simpler expressions for the spatial weights such as the inverse distance between two spatial units *i* and *j* (a) or the negative exponential function (b)[7].

$$(a)\begin{cases} W_{ij} = (d_{ij})^{-\beta};\ \forall\ i \neq j \\ \quad W_{ii} = 0 \end{cases}$$
$$(b)\begin{cases} W_{ij} = e^{-\alpha d_{ij}};\ \forall\ i \neq j \\ \quad W_{ii} = 0 \end{cases} \quad (7)$$

with α and β are fixed parameters.

Specification of the spatial weight matrix W is critical in spatial econometrics and can influence the significance of results. However, there is little guidance in the choice of the adequate spatial weights in empirical applications.

3.2. Spatial Lag Operator

In time-series context, values for neighboring observations can be facilely showed by using a lag operator (a backward-shift operator (y_{t-k}) or forward-shift operator (y_{t+k}) on the one-dimensional time axis (Anselin, 1988; Anselin & bera, 1997). This operator can shift the variable by one or more periods in time (k is the desired shift (or *lag*)). Instead, due to the various directions in which the lag can occur, there is no equivalent spatial shift operator in spatial econometrics (for more details see Anselin (1988)).

This problem is resolved by considering spatial weight matrix, that can link a variable at one point in space to the observations for that variable in different spatial units in the system. In fact, the key role of W may also be appreciated via the concept of spatial lag operator, it represents the weighted average of the values at neighboring locations. The spatial lag operator can quantify the effect of the dependent variable (Y) or, the explanatory variables (X) or the error term (ε) measured in other spatial units j than unit i on the dependent variable of unit i (Elhorst and Vega 2017).

Formally, a spatially lagged dependent variable is calculated as the product of a spatial weights matrix W with the vector Y. Thus, each element of the resulting spatially lagged variable can be written as following:

$$[W_y]_i = \sum_{j \in J} W_{ij} y_j \qquad (8)$$

This concept of spatial operator is particularly important because it introduces spatial autocorrelation into spatial econometric models. In general, spatial interactions are included into a spatial model by affecting a spatial lag operator to the dependent variable, or to the explanatory variables, or to the error term (Anselin *et al.* 2008). As a result, a wide range of spatial models can be specified.

[7] According to Anselin (2002) and Ertur and Kalidou (2005) these various kinds of spatial weights may be generalized by limiting the neighborhood for each spatial unit to a certain number (k) of locations where there is no interaction beyond that space (this matrix is called a "k-nearest neighbors".)

II. SPATIAL ECONOMETRIC MODEL SPECIFICATION

Spatial econometric model has recently used as a useful tool to study the spatial interaction effects and interdependence between spatial units. As documented by Elhorst (2010) spatial econometricians' way of thinking marks a sea change in 2007. Originally, spatial econometrics focused on one type of spatial interaction. Up to 2007, spatial econometricians were interested mainly in the spatial lag model also known as the spatial autoregressive model (SAR model) and the spatial error model (SEM model). The SAR model includes the spatial dependence through independent variable (spatially lagged dependent variable), while the SEM model includes spatial dependence through the errors term (Anselin 2001; Elhorst 2010; 2013). After 2007, there has been a growing interest in models containing more than one spatial interaction effect (for more details, see Elhorst (2010; 2014)).

In terms of model specification and estimation, the spatial econometrics literature has exposed an increase interest in the specification and estimation of spatial econometric models. Several regression models have been suggested to deal with the spatial dependence. There are two broad strands of the spatial econometrics literature: The first strand of the literature focuses on the spatial dependence models for cross-section data and the second strand of the literature focuses on spatial dependence models for panel data.

1. Linear Spatial Dependence Models for Cross-Section Data

In most empirical work, the standard approach is to start with the standard linear regression model (SLM) (it's a non-spatial linear regression model), and then to test if the model needs to be extended with spatial interaction effects (Elhorst 2010; 2013; 2013a; 2014). This approach is familiar as "the specific-to-general approach".

The standard linear regression model takes the form:

$$Y = \alpha\, I_N + X\beta + \varepsilon \quad (9)$$

Where:

- *Y is a (N *1) vector consisting of observation on the dependent variable for every unit in the sample (i = 1…, N)*
- *is a (N * 1) vector of ones.*
- *is a constant term parameter to be estimated.*
- *X is a (N * K) matrix of exogenous explanatory variables.*
- β is a (K * 1) vector of unknown parameters to be estimated.
- ϵ is an (N * 1) vector of disturbance terms, where ϵ i is assumed to be independently and identically distributed (iid) for all i with zero mean ($E(\varepsilon i)=0$) and variance σ^2 ($V(\varepsilon i)=\sigma^2$).

The non-spatial linear regression model is commonly estimated by Ordinary Least Squares (OLS): it is often labeled the OLS model. The non-spatial model is frequently em-

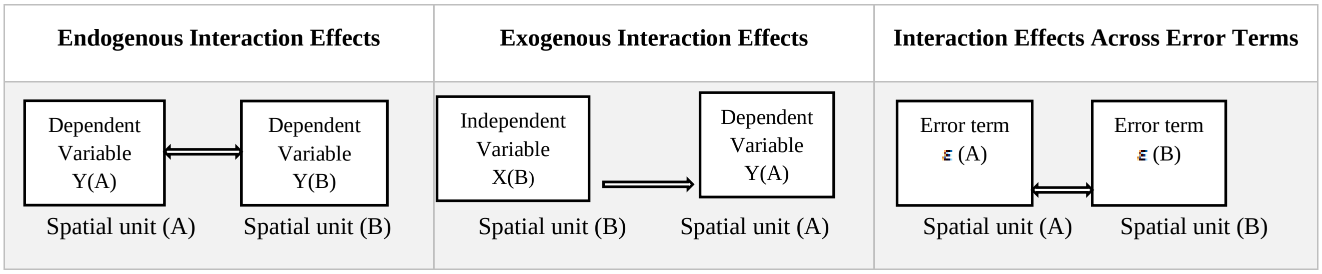

Fig. (1). Spatial Interaction effect.

Source: Adapted from Elhorst (2014).

ployed as a benchmark for comparisons with spatial models and as a diagnostic tool for model specification evaluation (Golgher and Voss 2015).

The opposite approach is to begin with a more general model containing all the type of interactions, this approach is known as “the general-to-specific approach”. Manski (1993) distinguishes three types of spatial interactions that may explain why an observation in a specific location may be influenced by observations at other locations: endogenous interaction effects, exogenous interaction effects and correlated effects (Fig. **1**).

- Exogenous interaction effects of independent variables (X): the decision of a spatial unit (A) to behave in some way, rely on the independent explanatory variables of the decision taken by other spatial units (B). The behavior of an individual is linked to the exogenous characteristics of the group.
- Interaction effects across error terms (ε) or correlated effects: the behavior of individuals belonging to the same group is similar because they face similar institutional environments or have similar individual characteristics.

An example may help to explicate this distinction. Consider the interdependence of probabilities of default within the banking system. There is an endogenous effect if, financial conditions of a bank rely on the state of the banking system. There is an exogenous effect if the probabilities of default depend on the bank's fundamentals (liquidity, solvency position, profitability). There is a correlated effect if homogeneous unobserved common macro-financial shocks strike banks at the same time.

Thus, the model with all types of interaction effects labeled the Manski Model or the General Nesting Spatial Model (GNS) (Elhorst 2013a; 2014; 2016) takes the form:

$$y = \rho WY + \alpha\, I_N + X\beta + WX\theta + u \quad (10)$$

$$u = \lambda Wu + \epsilon$$

Where

- WX the exogenous interaction effects among the explanatory variables.
- $W\mu$ the interaction effects among the disturbance terms of the different units.
- ρ is the spatial autoregressive coefficient, $|\rho| < 1$.
- λ is the spatial autocorrelation coefficient, $|\lambda| < 1$.
- θ is a (K*1) vector of fixed but unknown parameters to be estimated.
- W The spatial weights matrix, is a (N * N) positive matrix that describes the spatial arrangement of the units in the sample.
- The other variables and parameters are defined as in model (9).

According to Elhorst and Vega (2017), the spatial econometrics literature presents seven different types of static spatial econometric models. Fig. (**2**) summarizes a family of eight linear spatial econometric models that have been considered in the literature, among which are the OLS model and the GNS model. Table (**1**) (which is similar to part of a table presented in Elhorst and Vega (2017)) reports an overview and designations of these models. Each model can be obtained from the GNS model by imposing restrictions on one or more of its parameters.

An important development in the spatial econometrics literature is the growing attention for spatial spillover effects (Elhorst 2013; Elhorst and Vega 2017; LeSage and Peace 2009; LeSage 2011; 2014 and Herculano 2018). One of the main advantages of using spatial econometric models is that they offer the possibility of measuring and quantifying the magnitude of direct and indirect (spillover) effects[8]. In this context, LeSage (2014) points out that spatial regression models can be used to define the concept of a spatial spillover, estimate their quantitative magnitude and test for their statistical significance[9].

Until recently, numerous empirical studies use the coefficient estimates of a spatial econometric model to draw conclusions as to whether or not spillover effects exist. However, LeSage and Peace (2009) indicate that this may lead to erroneous conclusions. They suggest that “The partial derivative interpretation of the impact from changes to the variables from different model specifications” represents a more effective basis for examining spatial spillovers.

Spatial spillover effect is obtained from the reduced form of a spatial econometric model. It represents the marginal impact of a variation in one explanatory variable in a specific unit on the dependent variable values in another unit. It's an

[8] Standard econometric models restrict spillovers effects to be zero.
[9] This applies to cross-sectional models as well as to spatial panel data models (Elhorst, 2011).

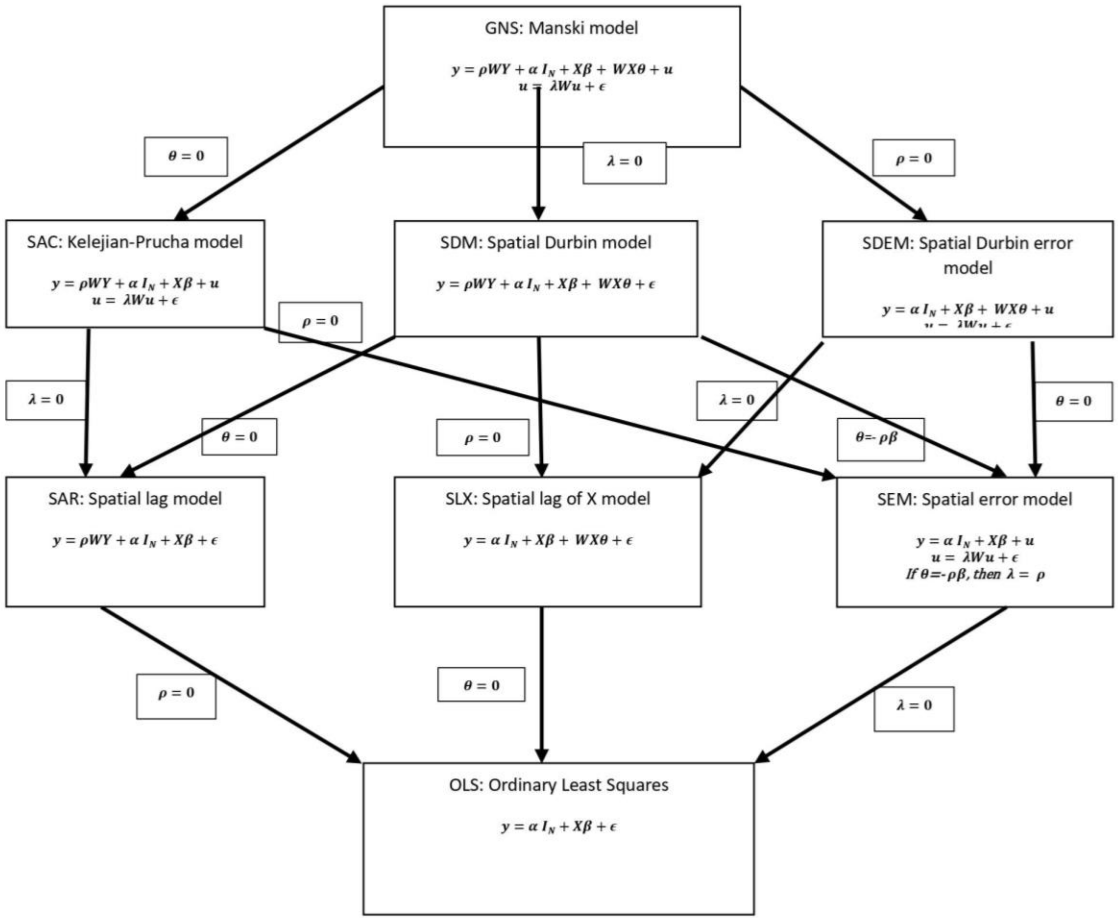

Fig. (2). The relationships between different spatial dependence models for cross-section data.
Source : Elhorst J.P.and Vega S. H., (2013): "On spatial econometric models, spillover effects, and W"

Table 1. Spatial Econometric Models with Different Combinations of Spatial Lags.

Spatial Econometric Model	Abbreviation	Number of Spatial Lag (s)	Type of Spatial Lag
Spatial autoregressive model (the spatial lag model)	SAR	1	*WY*
Spatial error model	SEM	1	*Wu*
Spatial lag of *X* model	SLX	1	*WX*
Spatial autoregressive combined model(the SARAR or Cliff-Ord model or Kelejian-Prucha model)	SAC	2	*WY, Wu*
Spatial Durbin model	SDM	2	*WY, WX*
Spatial Durbin error model	SDEM	2	*WX, Wu*
General nesting spatial model	GNS	3	*WY, WX, Wu*

Source: adapted from Elhorst and Vega (2017).

addition to the direct effect which captures the marginal impact of a change to one explanatory variable in a specific unit on the dependent variable of that unit itself (Elhorst and Vega 2017). Thus, for a spatial regression model, the direct effect is when a change in the explanatory variable of a particular unit affects the dependent variable of that particular unit itself while the indirect effects (spatial spillover) is when this same change affect the dependent variables in other units.

The direct and indirect effects corresponding to the different spatial econometric models presented in Fig. (**2**) are reported in Table (**2**). Elhorst (2010; 2013; 2014), Elhorst and Vega (2013), LeSage (2014), Golgher (2015) and LeSage and

Table 2. Direct and Spillover Effects Corresponding to Different Model Specifications.

Spatial Econometric Models	Direct Effect	Spillover Effect	Flexibility Spatial Spillovers
SAR	Diagonal elements of $(1-\rho W)^{-1}\beta_K$	Off-diagonal elements of $(1-\rho W)^{-1}\beta_K$	Constant ratios
SEM	β_K	0	Zero by construction
SLX	β_K	θ_K	Fully flexible
SAC	Diagonal elements of $(1-\rho W)^{-1}\beta_K$	Off-diagonal elements of $(1-\rho W)^{-1}\beta_K$	Constant ratios
SDM	Diagonal elements of $(1-\rho W)^{-1}[\beta_K + W\theta_K]$	Off-diagonal elements of $(1-\rho W)^{-1}[\beta_K + W\theta_K]$	Fully flexible
SDEM	β_K	θ_K	Fully flexible
GNS	Diagonal elements of $(1-\rho W)^{-1}[\beta_K + W\theta_K]$	Off-diagonal elements of $(1-\rho W)^{-1}[\beta_K + W\theta_K]$	Fully flexible

Source: adapted from Elhorst and Vega (2013; 2017)

Peace (2009) analyze in more details direct and indirect effects of the different spatial econometric models.

2. Linear Spatial Dependence Models for Panel Data

Panel data refer to a cross-section of observations iterated over several time periods. Spatial panels can be defined as data including time series observations of a number of spatial units such as zip codes, municipalities, regions, states, countries, etc. (Elhorst, 2009). Spatial panel models have panel data structures to represent spatial interactions through spatial units and over time. In recent years, the spatial econometrics literature has manifested an increasing interest in the specification and estimation of panel data models with cross-sectional dependence. Recent contributions are, for example, Elhorst (2001; 2003; 2009; 2010; 2011; 2013a; 2014), Anselin *et al.* (2008), Debarsy and Ertur (2010), Lee and Yu (2010a; 2010b; 2010c; 2010d; 2014), Millo and Piras (2012), He (2015), leSage (2015), etc.

Spatial panel data models are of great interest; this can be explained by some motivations (Elhorst 2003; 2009; 2011; 2013a; 2014):

- Panel data allow for the identification of more complicated behavioral hypotheses (including effects that cannot be treated using pure cross-sectional data),
- Panel data control for unobservable heterogeneity,
- Panel data are usually more informative,
- They contain more variation and less collinearity among the variables,
- Panel data increase the degrees of freedom, and consequently augment efficiency in the estimation.

Anselin *et al.* (2008), Elhorst (2013a) and Elhorst (2014) explicate how the traditional cross-sectional models can be extended to panel data models. The general nesting spatial model (GNS model), presented in equation (9), can be extending to a space-time model for a panel of N observations over T time periods by adding a subscript t ($t = 1 \ldots T$) to the variables and the error terms of that model:

$$Y_t = \rho W Y_t + \alpha\, I_N + X_t\beta + W X_t\theta + u_t \quad (11)$$

$$u_t = \lambda W \mu_t + \epsilon_t$$

As stated by Elhorst (2014), "This model can be estimated along the same lines as the cross-sectional model, provided that all notations are adjusted from one cross-section to T cross-sections of N observations"[10]. Furthermore, other spatial econometric models can be obtained from this GNS model by imposing restrictions on one or more of its parameters: OLS, SAR, SEM, SLX, SAC, SDM, and SDEM (These restrictions are similar to those indicated in Fig. **2**).

However, the main objection to this model is that it does not account for spatial and temporal heterogeneity (Elhorst 2003; 2009; 2014), this may increase the risk of obtaining biased estimation results. One remedy is to introduce two variables intercepts. One is a spatial specific effects variable to control for all time-invariant variables (whose omission could bias the estimates in a cross-sectional analysis). The other is a time-period specific effects variable (whose omission could bias the estimates in a time-series analysis) to control for all spatial-invariant variables.

The space–time model in (10) reads as:

$$Y_t = \rho W Y_t + \alpha\, I_N + X_t\beta + W X_t\theta \cdot \xi_t I_N + u_t \quad (12)$$

[10] Contrary to the classic panel data where the data is stored first by spatial unit and then by time, in the spatial panel data, the data is sorted first by time and then by spatial unit.

$$u_t = \lambda W \mu_t + \epsilon_t$$

Where μ the spatial specific effects and ξ the time-period specific effects.

The spatial and time-period specific effects can be estimated as fixed effects or as random effects. In the fixed effects model, a dummy variable is introduced for each spatial unit or for each time period. While in the random effects model, these two effects (μ_i, ξ_t) are treated as random variables, they are supposed to be independently and identically distributed (with zero mean and variance σ_μ^2 and σ_ξ^2 respectively). Equally, it is assumed that all the random variables (μ_i, ξ_t, and ϵ_t) are independent of each other (Elhorst 2014).

In the spatial econometrics literature, there are static as well as dynamic spatial panel data models. To illustrate the difference, the Spatial Durbin Model (SDM) which, becomes more broadly used in applied research (Elhorst and Vega 2013), will be studied.

In the one side, a static spatial panel model does not hold variables that allow for time dependence (LeSage 2015). The estimation of this models is widely discussed in Elhorst (2003; 2010; 2014), Lee and Yu (2010; 2010a; 2010c); etc.

The Spatial Durbin Model (SDM) for panel data is obtained when imposing the parameter restrictions λ = 0 on Equation (10), it reads as :

$$Y_t = \rho W Y_t + \alpha\, I_N + X_t \beta + W X_t$$
$$\xi_t I_N + u_t \quad (13)$$

Static spatial panel data models can be extending to dynamic spatial panel data models. To do this, an immediate approach is to add the time lag term as an explanatory variable (Lee and Yu 2010). The specification and estimation of dynamic spatial panel data models are provided by Elhorst (2011; 2013a), Lee and Yu (2010), Anselin *et al.* (2008), etc. Elhorst (2011) presents a generalized dynamic model in space and time that generalizes various simpler models considered in the literature. This model might consider time lags of the variables Y_t and WY_t, time lags of the variables X_t, WX_{t-1} and time lags of the error terms U_t and WU_t. Elhorst (2014) notices that an error term lagged in both space and time is not included in such model since it is uncommon in the literature). However, as stressed by Anselin et al. (2008) and Elhorst (2011), the parameters of this generalized model suffers from identification problems. Elhorst (2011) presented an overview of the main restrictions considered in the literature and exposes a taxonomy of dynamic models in space and time.

To make The Spatial Durbin Model (SDM), presented in equation (13), dynamic, one might add time lags of the variables Y_t and WY_{t-1} (Elhorst 2011; Lee and Yu 2010; Debarsy *et al.* 2012). This equation can be rewritten as:

$$Y_t = \tau Y_{t-1} + \rho W Y_t + \eta W Y_{t-1} + \alpha\, I_N +$$
$$X_t \beta + W X_t \theta + \mu + \xi_t I_N + u_t \quad (14)$$

The dynamic spatial Durbin model include a dependent variable lagged in time, a dependent variable lagged in space, a dependent variable lagged in both space and time, spatial-specific and time-period-specific effects.

3. Estimation and Model Comparison

3.1. Spatial Econometric Models Comparison

Some of the spatial econometric models recorded in figure (2) are well known and often considered in econometric-theoretic and empirical research, while other models are not. The spatial Durbin error model (SDEM) is the best example. Elhorst and Vega (2013) notice that the SLX model and the SDEM model are usually overlooked. Elhorst (2013) states that the estimation of the SLX model does not post any econometric problems, so this model is generally not part of the toolbox of researchers concerned in the econometric theory of spatial models. Likewise, the parameters of the GNS model that incorporates all interaction effects are unidentified and as noticed by Elhorst *et al.* (2016), this model is practically never utilized in empirical applications.

Moreover, Elhorst (2013) stresses that “there is a large gap in the level of interest in different types of interaction effects between theoreticians and practitioners”. Theoreticians are mainly interested in the SAR, SEM and the SAC model because of the econometric problems appearing with the estimation of these models. LeSage and Peace (2009) outline that the SDM model produce unbiased coefficient estimates. In addition, when a study focuses on spatial spillover effects, Elhorst and Vega (2017) suggest that the SLX model is the simplest model producing flexible spatial spillover effects.

The spatial econometrics models could take different formats, thus, as mentioned by Asgharian *et al.* (2013) the selection of model specification is not ineffectual. There are two strategies to find the spatial econometric model that best describes the data: the general-to-specific approach or the specific-to-general approach. Whereas, LeSage and Pace (2009) argue that the SDM model is the best point of departure, Elhorst (2010) proposes to estimate the OLS model and then test if it can be extended to the spatial lag model or the spatial error model. While, Elhorst and Vega (2013) propose to select the SLX model as point of departure.

In this respect, there is various statistical tests developed in the spatial econometrics literature to distinguish between alternative spatial model specifications. For example, the Moran’s *I* statistic is applied to verify whether spatial autocorrelation should be considered (spatial versus non-spatial model). The classic Lagrange-Multiplier (LM) tests (Anselin 1988) and the robust LM tests (Anselin et al. 1996) are used to verify whether the SAR model or the SEM model is more appropriate to describe the data. These tests are based on the residuals of the OLS model. Anselin *et al.* (2006) extend these tests to the case of spatial panel. These tests are illustrated in Elhorst (2014) using Anselin’s (1988) cross-sectional dataset of 49 Columbus, Ohio neighborhoods and using a panel dataset on cigarette demand from Baltagi (2001). There are also others test such as the Wald test (W), the Likelihood Ratio test (LR) (for an overview of these tests see (Anselin 1988a), the J-test and the Bayesian comparison approach.

3.2. Methods of Estimation

To estimate spatial models, a growing number of methods have been advanced in the spatial econometrics literature, including the maximum likelihood (ML), quasi-maximum likelihood (QML), instrumental variables (IV), generalized method of moments (GMM), and Bayesian Markov Chain Monte Carlo methods (Bayesian MCMC) (Elhorst 2009; 2010; 2011; 2014; LeSage 2014). Standard statistics packages do not include estimation routines for spatial econometric models. These models can be estimated using various Software packages and/or routines such as GeoDa and OpenGeoDa software, MATLAB-based econometrics toolbox develcopped by LeSage (2010), STATA, R.

III. APPLICATION OF SPATIAL ECONOMETRICS IN FINANCE

Spatial econometrics has been broadly used in applied geographic and regional science studies and has percolated into the field of finance more recently (Arnold *et al.* 2013; Debarsy et al. 2018; Foglia and Angelini 2019; Blasques et al. 2016). The reason why it is not yet very popular in financial applications might lie in the difficulty of finding a correct measure of economic distance. Weng and Gong (2016) argue that "The applications of such models in economics and finance are not yet very popular, due to the difficulties in constructing the spatial weight matrices in the context of financial markets". Equally, Catania and Billé (2017) indicate that "In finance the choice of weighting matrix is not easy, mainly due to the immateriality of the notion of distance". Therefore, it is not evident how distance should be gauged and the challenge will be how to describe contiguity in the field of finance.

Spatial econometrics has its roots in the analysis of geography, so the usual tradition in choosing the spatial weight matrix is based on geographical distance. However, there is no clear reason for why spatial distance should need to be restricted to geographic distance.

In finance, distances between neighbors are not limited to only geographic distances but can represent financial (or economic) distances as well[11]. As mentioned by Fernandez-aviles *et al.* (2012), in financial markets, financial distance is an alternative to the physical distance and, it represents a measure of pairwise financial closeness between spatial units.

In this section, a brief survey of some recent applications of spatial econometrics techniques in finance will be explored.

1. Application of Spatial Econometrics to Study the Co-movements in Stock Markets

The developments of spatial econometrics have provided an excellent tool for performing analyses of the linkages that are significant for the co-movements of financial markets.

Fernandez (2011) explores the possibilities of using the notion of spatial dependency in the field of finance. She formulates a spatial version of the capital asset pricing model (S-CAPM) in order to test for spatial dependency in a panel of over 100 firms situated in Brazil, Chile, and Mexico over the period 1997–2006. In order to capture the spatial interaction between neighboring firms, Fernandez (2011) considers four financial indicators (ratios) such as market capitalization, the market-to-book ratio, the debt maturity ratio and the dividend yield. She founds that there exist spatial effects in general, but when looking at each country separately, the evidence is mixed.

Fernández-Avilé *et al.* (2012) investigate if the first law of geography (Tobler, 1970) is pertinent to financial contexts in general and to stock exchange returns in particular. Using spatial techniques, they analyze to what extent various linkages between countries impact the degree of stock market co-movements. These authors examine daily data on 17 market returns over the period between January 2002 and March 2010. They present the spatial distance as both physical and financial distance. They point out that the dependencies between market returns are unrelated to physical distances but highly related to financial linkages, as measured by foreign direct investment (FDI) ties.

Arnold *et al.* (2013) propose a spatial autoregressive (SAR) model in order to analyze spatial dependencies in the Euro Stoxx 50 returns for the period from 2003 until 2009. Their model allows for distinguishing between three different kinds of spatial dependence. The first type is a general dependence affecting all stocks in the same way. The second type represents dependencies between different industrial branches (it captures dependencies between firms that belong to the same industrial branch). The third type accounts for local dependencies (firms belonging to the same country should exhibit similar behavior). They conclude that the spatial approach seems to be more suitable to estimate and measure risk in portfolio management than the standard approaches such as a sample covariance matrix or a factor model. Considering the model from Arnold *et al.* (2013), wield (2013) proposes a CUSUM-type test for time-varying parameters in order to account for structural breaks. He concludes that considering structural changes can lead to further accurate risk forecasts.

Similar evidence has been developed by Asgharian *et al.* (2013), who use the Spatial Durbin Model (SDM) to study the co-movements in stock market return across 41 countries between January 1995 and December 2011. Asgharian *et al.* (2013) consider that stock market integration of different countries is related to their closeness as defined by a variety of financial and economic integration measures. They use various measures that combine geographical distances (distance between the capital cities for every pair of countries) and financial distances such as exchange rate, expected inflation, purchasing power parity, interest rate, bilateral trade, bilateral FDI. They find that bilateral trade proves to be best adapted to capture co-movements in returns.

Recently, Baumöhl *et al.* (2018) analyze volatility spillovers and its determinants among 40 developed, emerging and frontier stock markets over the period spanning from January 2006 to December 2014. Using spatial model incorporating several exogenous characteristics, they document the presence of significant temporal proximity effects and a highly

[11] Economic distance and financial distance are used interchangeably and refer to the explained measure of distance in the field of finance

spatial dependence among markets. Baumöhl *et al.* (2018) show that the most relevant determinants of volatility are market size, liquidity and economic openness and, that networks of volatility spillovers can be used to capture the interconnectedness of individual stock markets.

Zhang *et al.* (2019) analyze the spatial return spillover among G20 financial market from 2006 to 2017. They develop the multidimensional spatial autoregressive panel model (SAR). Their results show that it is essential to add a spatial weight matrix to the econometric model in order to capture the multidimensional spatial spillover effects among stock markets.

Chen and Jin (2020) consider a dynamic spatial panel data model to study the industry risks in China's stock market. They choose the Shanghai-Shenzhen 300 industry indices from Shanghai Stock Exchange and Shenzhen Stock Exchange from January 2005 to August 2018. The authors analyze the risk spillovers by considering various transmission channels. They found that the spillover effects exist and are persistent, and that the information channel outperform the real linkage channel.

Tissaoui and Zaghdoudi (2021) investigate the co-movement between the U.S. financial market and the Euro-Asian financial markets. Using a spatial least square regression, they analyze seven international implied VIX indexes from April 2010 to February 2019. They detected a significant spatial spillover effect and a dynamic interaction between the U.S. market and European and Asian markets. Equally, they highlight a significant transmission of risk over time.

Some further interesting contributions can be found in Eckel *et al.* (2011); Durante *et al.* (2014); Tam (2014); Schmitt *et al.* (2015); Gong and Weng (2016); Chluun (2016); Bera and Kececi (2016); Weng and Gong (2016), Zhu and Milcheva (2015; 2016; 2018), Catania and Billé (2017), Selan and Kalatzis (2017), Chulia *et al.*, (2017), Kutzker and Wied (2019).

2. Application of Spatial Econometrics to Study Contagion in Sovereign Bond Markets

Spatial econometrics techniques can be utilized to shed light on the extent of contagion across sovereign bond market. For instance, Dell'erba *et al.* (2012) investigate the issue of spillovers in the sovereign bond market for 24 emerging economies over the period between 1995 and 2010. They measure the interconnectedness across countries using various weighting matrices based on geographical proximity, business cycle synchronization, trade linkages, financial linkages, sovereign rating status and institutional similarity. They find strong evidence of spillovers across emerging market economies. Another example is Umberto (2014) who explores the presence of contagion effects in the Euro Area Sovereign Bond Market. The country sample includes ten European countries over the period from 2007 to September 2013 and the economic distance between pairs of countries is quantified using correlations of the stock markets. As a result, he documents the presence of contagion in the Eurozone countries.

Using a large-scale database including 41 advanced and emerging economies over the period 2008 to 2012, Debarsy *et al.* (2018) explain international spillovers of sovereign bond spreads and assess the role of transmission channels. They estimate a spatial dynamic panel data model and take into account both real linkages and informational transmission channels such as bilateral trade flows, debt-to-GDP similarity, deficit-to-GDP similarity, the government stability index closeness, socioeconomic proximity index. Their results reveals that these channels are all pertinent to explain risk transmission, however, the informational channel is of highest importance.

Furthermore, Asgharian *et al.* (2018) extend the vector autoregressive (VAR) model to a spatial VAR model in order to study the importance of cross-border asset holdings for the co-movement of government-bond yields. Their sample comprises eleven Euro-area countries and the US from December 2001 to December 2012. To describe the relative closeness of the countries to each other, they use cross-border bank lending and the cross-border holdings of debt and equities. They show that cross-border holdings of long-term debt and bank lending are significant for the interdependence in yield curves among Euro-area countries while the short-term debt and equity holdings are unimportant.

Lately, Asgharian *et al.* (2020) investigate the co-movements of international stock and bond markets. They develop a structural multivariate spatial regression model and measure countries' proximity to each other using geographic neighborhood and bilateral trade. They found a significant spatial dependence between countries' bond returns which is smaller than that of stock returns.

3. Application of Spatial Econometrics to Study Contagion in Sovereign Credit Default Swap Markets

Sovereign Credit default swaps (CDS), a specific innovation in financial markets, move into the spotlight of financial markets since the global financial crisis, and especially during the European sovereign debt crisis. They function as insurance contract that is originally created to protect investors against the default of sovereign entity. Sovereign Credit Default Swap spread are designed to trade and manage credit risks. They are often perceived as a better advanced indicator for approximating sovereign credit risk (Delatte *et al.* 2010; Andenmatten and Brill 2011; Longstaff et al. 2011; Turguttopbaş 2013; Augustin 2014; Broto and Pérez-Quirós 2015; Bedendo and Colla 2015; Stolbov 2016; Bocola 2016; Galariotis *et al.* 2016; Bai and Wei 2017). Recently, the application of spatial econometrics framework seems particularly well-suited for examining financial contagion in the sovereign CDS markets.

In this context, Blasques *et al.* (2014) and Blasques *et al.* (2016) apply spatial econometrics tools to report the role of contagion among eight European countries over the period 2009–2014, covering the Eurozone debt crisis. On the one hand, Blasques *et al.* (2014) extend the well-known static spatial lag model for panel data to a new model quantifying the time-varying cross-sectional dependence. On the other hand, Blasques *et al.*, (2016) extend the well-known static spatial Durbin model by a time-varying spatial dependence parameter between sovereigns. In the two study, they measure the structure of the economic neighborhood between the sovereign CDS spreads using cross-border lending data.

They document a strong time-varying degree of spatial dependence in the European sovereign CDS spreads. They also record a downturn in spatial dependence towards the end of 2012.

A seminal methodological contribution is given by Zhu (2018) who combines a spatial model with a panel VAR model to obtain a spatial vector autoregressive (SpVAR) model. Using data on CDS spreads, Zhu (2018) explores the cross-border spillovers of sovereign, banking and corporate default risks among eleven Eurozone countries over the period between January 2008 and December 2013. Various weighting matrices based on trade integration, countries' bilateral trade flows and bilateral bank claim exposures are used. The results show significant spatial dependencies across the eleven countries and the three sectors and that, shocks to the banking sector have the most critical role in the crisis transmission.

Finally, Mwamba and Manguzvane (2020) investigate the extent to which geographical proximity and international trade affect the stability of African sovereign-debt markets. They estimate a spatial Durbin model using sovereign CDS spreads of Six African countries from 2008 to 2018. Their results show that both country's macroeconomic fundamentals and contagion from other countries influence its likelihood of default and, that trade linkages are a strong transmission channel for contagion risk.

4. Application of Spatial Econometrics to Study Credit-Risk Propagation in Financial Institutions

Recent studies propose a new bank systemic risk measure based on spatial econometrics approach to take into consideration the network structure of the financial system. Keiler and Eder (2015) apply a static version of the spatial autoregressive (SAR) model on a sample of 15 important financial institutions over the period 2004 to 2009 to measure, quantify and model the systemic risk within the financial system. Their spatial econometric approach allows for a decomposition of the credit spread into three components: a systemic, a systematic and an idiosyncratic risk component. The degree of proximity between financial institutions is approximated by the equity correlation between two company. Results indicate a considerable risk spillover across the financial institutes in the sample.

Similarly, using the spatial autoregressive (SAR) model, Herculano (2018) studies the evidence of contagion in explaining financial distress within the US banking system. They use a Bayesian spatial autoregressive (SAR) model and examine a panel of a considerable number of banks from 1990 to 2018. He discovers that contagion contributes to the augmentation of distress in the banking system and there is a significant heterogeneity where some institutions are systemically more important that other.

Calabrese *et al.* (2017) describe the contagion effects in the Eurozone banking system using a binary spatial autoregressive model. They document evidence of a relatively high level of systemic risk due to contagion effect during the European sovereign debt crisis.

Foglia and Angelini (2019) analyze Eurozone Banking Systemic Risk. They use a spatial dynamic panel model which is the time-varying SAR model following the model of Blasques *et al.* (2014). They study the contagion effects using CDS spread of 22 listed Eurozone banks from December 2008 to February 2017. To build the spatial weighted matrix, Foglia and Angelini (2019) utilize the Financial Claim matrix and the stock correlation weighting matrix. They document a strong spatial dependence in the evolution of CDS spread across the Eurozone banks and that monetary policy were effective in reducing systemic risk.

More recently, observing the CDS market from 2009 to 2017, Foglia *et al.* (2020) analyze the evolution of financial contagion among 22 major Eurozone banks. The authors use a dynamic spatial Durbin model and highlight how contagion spreads through physical and financial linkages between banks. They document a significant evidence of the presence of credit risk spillovers in the CDS markets and, that equity market dynamics of "neighboring" banks are valuable factors in risk transmission.

5. Application of Spatial Econometrics to Explore the Drivers of Financial Stress and to Identify Risk Spillover Channels

Nowadays countries are becoming more integrated. The 2007 Global Financial Crisis has showed that the economy of one country is not independent of the economies of others. In this context, spatial econometric model has lately appeared as a practical tool to identify risk spillover channels. For instance, Jing *et al.* (2017) model spillover and interdependence effects to analyze the propagation of financial turbulence via capital flows, trade, and distance channels. Using spatial econometric techniques and a sample of 40 countries from 2003 to 2010, they show that interdependence and spillover effects should be jointly studied. Capital flows channel surpasses trade and distance channels in capturing propagation of financial turbulence. Also, they estimate the direct and indirect effect and demonstrate that the marginal effects of macroeconomic variables on financial turbulence are different during crisis period and tranquil period.

Begüm and Özlem (2021) explore the determinants of financial stress and analyze the impact of the spatial linkages between 13 emerging economies from1996 to 2016. They observe a strong interaction of financial stress. Economic growth, current account balance over GDP, global risk and geopolitical risk are the most important drivers of financial stress.

Huang and Liu (2021) outline the importance of considering various channels when analyzing sovereign risk spillovers. They use a sample formed by 41 advanced and emerging economies during 2004 and 2019. They find that sovereign risk spillover channels are different in various periods of crisis. Real linkages and information channel play a major role during the full sample period. Business connections have an effect only during the financial crisis period.

Finally, by focusing on different transmission mechanisms, Jiang *et al.* (2022) investigate the contagion of the US subprime crisis across the world. They apply spatial analysis on a sample of 36 countries from 2002 to 2018. They find that countries with bad fundamentals (higher inflation, lower exchange rates, lower current accounts) tend to suffer more from financial crises. International trade and financial linkages play the most prominent roles in the rapid spread of financial crises around the world than macroeconomic fundamentals and political similarities.

CONCLUSION

Spatial econometrics is a subfield of econometrics. It is a relatively young and growing field that aim at taking into account the spatial dependence among sample observations. This indicates that what happens in one unit of analysis is linked to what happens in neighboring units.

The spatial econometrics literature has exposed an increase interest in the specification and estimation of spatial econometric models. Recent years have seen a virtual explosion in the application of these models in various fields. It has been advanced quickly and moved from the margins (applied geographic and regional science studies) to the mainstream (economics): It has recently been applied in empirical finance.

Spatial econometrics techniques can be used in various financial topics. They can be employed to investigate the comovements of international asset markets and, to identify risk spillover channels. They can be utilized to further shed light on financial contagion across both Sovereign Bond Markets and Sovereign Credit Default Swap Markets. Also, this framework appears particularly well-suited for analyzing the financial institutions credit-risk propagation.

Spatial econometrics has been criticized by numerous economists. A financier who try to use spatial econometric methods may find the spatial econometrics literature confusing. For instance, one obstacle is the large number of alternative model specifications discussed in the literature. Different model specifications signal different spatial correlation structures, that may be contrastive to the economic theory behind the interaction model, also, some model specifications did not have a firm foundation in economic theory but have been driven by data-analytic considerations. Another considerable weakness of spatial econometric models is that the specification of the spatial weights matrix W is often ambiguous. Although Weight matrices have a key role and that estimates of spatial models are sensitive to specification of these matrices, there is a little guidance in the choice of the correct spatial weights in the spatial econometrics literature.

In this papers, the peculiarities of spatial econometrics are descripted. Hopefully, a financier with little experience with these techniques can benefit from the presentation of simple principles of spatial models set forth here.

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