

Teaching¹ Economics Students about Different Models for Dealing with Uncertainty and Risk Besides the Standard Capital Asset Pricing Model (CAPM) and the Subjective Expected Utility (SEU) Model

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Abstract: There has been a great deal of uncertainty (doubt) and fear about how the COVID-19 corona virus would impact the world's economies in the future. This fear of the future would explain the manner in which individuals and countries have responded to the outbreaks by buying gold and/or other "hard" assets, which decision makers have great confidence in.

In times of uncertainty (doubt), holding gold is a reliable and dependable way of combatting the likely impact of uncertain events on future events. The COVID-19 virus has generated a great deal of fear regarding the economic effects of the virus on the economy. Holding hard assets would allow the holder of such assets to feel safer and more secure about their ability to successfully deal with and/or wait out such events.

We argue that undergraduate students would be better prepared for decision making in the real world after they graduate if the standard approach taken in microeconomic courses based on risk assessment alone was supplemented by alternative treatments that do not model decision making as taking place only under the assumption of additivity and linearity as is made in CAPM and SEU. It is important to teach students how to modify their probabilities to transform them into decision weights which (a) consider uncertainty, but (b) simplify to probabilities if the uncertainty should diminish substantially in the future. This is accomplished by using Tversky-Kahneman's Cumulative Prospect Theory and Keynes's Conventional Coefficient model from the *A Treatise on Probability*.

Keywords: Linear Risk, Nonlinear, Uncertainty, Investing, Transforming precise probability into imprecise probability.

INTRODUCTION: UNCERTAINTY AND THE CORONA VIRUS

The Dow 30 reached its then record high of 29,551.42 on Feb. 12, 2020. From that day on, as concerns about the spread of the novel corona virus started to intensify, through early March, when President Trump announced a nationwide shutdown on March 12, 2020, the Dow fell by huge amounts daily. The maximum point drop in the DOW 30 between its February 12, 2020 closing high of 29,551 and its close on March 18th at 19,908 occurred on March 16th, 2020. The drop in the Dow that day was 2,997.10 points.

It continued to fall through March 18th, 2020, when it closed at 19,903. The low point for the DOW was reached on Mar. 23 when it hit 18,591.93. A massive 1.8- 2.0 Trillion-dollar Government stimulus package was passed in early April, 2020 that put a halt to the decline in the DOW and allowed a recovery to start. During the same time period the price of gold rose from \$1567 on March 23rd, 2020 to \$1881 on December 18th, 2020.

There appears to have been a great deal of uncertainty (doubt) and fear about how this particular corona virus would end up impacting the world economy in the future. This fear of the future would explain the manner in which individuals and countries have responded to the outbreaks by buying gold and/or other "hard" assets about which decision makers have great confidence in.

We can't find any other situation that parallels the manner in which national and international reactions have taken place on a worldwide level unless one wants to compare the coro-

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na virus to the outbreak of the Black Death (Bubonic Plague), which killed 40 % of Europeans between 1346 and 1358 or the Spanish (which actually originated in China) Flu of 1918-1921, which is estimated to have killed 40-100 million worldwide. There was no organized economic response to these past events when they are compared to the economic responses that took place in the USA and the rest of the world in the year 2020.

Uncertainty resulted from the following two pandemics that occurred in the 20th centuries. The first was the 1957–1958 influenza pandemic, also known as the Asian flu. It was a global pandemic of influenza A virus, subtype H2N2, which originated in Guizhou, China. A 2016 study estimated the number of deaths caused by the 1957-1958 pandemic at 1.1 million worldwide. It was first reported in Singapore in February 1957, Hong Kong in April 1957, and in coastal cities in the United States in the summer of 1957. The estimated number of deaths was 116,000 in the United States.

The second was the Hong Kong flu, also known as the 1968 flu pandemic. This outbreak occurred in 1968 and 1969 and killed an estimated 1–4 million people globally. It was caused by an H3N2 strain of the influenza A virus, which is descended from H2N2 through antigenic shift, a genetic process in which genes from multiple subtypes are reassorted to form a new virus. (Jester, et al, 2020; Chang, 1969).

There was no major response in the USA to either of these pandemics with respect to changes in the DOW or business lockdowns or major fiscal stimulus programs. The same holds with respect to outbreaks of HIV in the 1980's and 90's, as well as SARS, MERS, and the EBOLA virus in Africa in the 21st century.

The novel corona virus appears to have one very major difference from these earlier flu pandemics or outbreaks discussed in the paragraph above. That difference is that 50% to 60% of infected individuals never have any significant symptoms when they come down with the novel corona virus. This means that they appear to be perfectly healthy, but they can still infect other people. We believe that this trait of the novel corona virus explains the great negative reaction to it, which generates fear and doubt that makes risk calculations unreliable and inaccurate. It is the uncertainty involved in dealing with individuals who look perfectly healthy, but who are still highly contagious, that is a major problem.

Since the time of the original outbreak of Covid-19 in Wuhan, a city of 11 million in the Hubei province, which is the 9th largest city in China, in November–December, 2019, uncertainty has been somewhat reduced. This would allow risk calculations by decision makers that would be more reliable. This is due to the large increase in knowledge about the basic nature of the virus. Effective vaccines and treatments have been created that have appeared to many to have reduced the impact of the virus, although the swift and unexpected appearance of the omicron variant of the virus in December, 2021 and January, 2022, with infection rates that far dwarfed all earlier versions of the virus, leads to the conclusion that the virus's ability to mutate over time can still lead in the future to unexpected break out of new variants, such as the potential deltacron variant that combines the severity of the delta II variant with the infectiousness of the omicron vari-

ant. Such an occurrence would increase uncertainty and reduce the accuracy of risk assessments

Currently, the standard microeconomic classes at the undergraduate level generally only teach the Capital Asset Pricing Model (CAPM) and/or the Subjective Expected Utility (SEU)-Expected Value (EV) models.

Two alternative approaches, that are based on transforming linear and additive probabilities into nonlinear and non-additive decision weights, are Keynes's 1921 conventional coefficient of weight and risk, $c=c(p)$, and Tversky and Kahneman's Cumulative Prospect Theory of 1992, which uses a transformation function, $\pi=\pi(p)$.

As regards Keynes's c coefficient, we will take J M Keynes's definition of uncertainty, that he gave in 1936 in his *General Theory* on page 148, as the definition we intend to use in our paper. Keynes's definition of uncertainty is that it is an inverse function of the evidential weight of the argument, w , which can be more accurately characterized as the amount of relevant and reliable data, information and evidence that is available to a decision maker, faced with a particular problem, event, outcome or occurrence at a particular time, to base his decision on. This would correspond to the heuristic of availability used in Kahneman and Tversky (1992) approach. Uncertainty will be high if citizens perceive that the experts themselves have doubts about how to proceed with specific, concrete steps, plans or remedies to deal with the outbreak of the novel corona virus. The ability to deal with the corona virus is the problem which we have selected as the decision problem that would be faced by students in an undergraduate class, with an emphasis placed on the word novel.

Keynes (1921) argued that uncertainty was difficult to measure statistically in an exact, precise fashion. He suggested the use of inexact and approximate methods that used upper and lower bounds or used the median or mode instead of the mean. For instance, Mandelbrot (2004), whom we will not consider further, as his approach would be taught to graduate students in economics, suggested that one needed to interpret the time series data as coming from distributions that did not have a finite mean and variance, like the Normal distribution (the data approximated distributions, like the Cauchy, that had infinite variance, which means the tails are long and thick, as opposed to the Normal distribution, where the tails are short and thin). The assumption of normality underlies the neoclassical Capital Asset Pricing Model (CAPM). It uses the standard deviation as a measure of risk. However, once it is observed that movements far greater than plus or minus three standard deviations (for instance, a change greater than + or - 2.55% in the DOW) around the mean were being observed, one could interpret such movements to students as leading to the conclusion that there is uncertainty occurring, as opposed to risk. The greater the number of standard deviations occurring, demonstrated by the extreme buying/selling behavior being engaged in, would lead a decision maker to consider that the normal distribution was not reliable to base future projections on. Mandelbrot uses the terminology of "wild" risk versus the neoclassical conception of "mild" risk, as used in the CAPM model, to argue against a modeling approach based on normality. See Mandelbrot and Hudson, 2004.

Tversky and Kahneman, like Keynes, reject the standard approach to decision making, based on precise, linear probabilities, in favor of using weighting functions that transform the linear probabilities into nonlinear decision weights. This accounts for the role that incomplete information and data, hence doubt and/or uncertainty, plays in decision making besides the usual representation of a problem being able to deal only with considerations of risk. Kahneman and Tversky use the weighting function $\pi=\pi(p)$ while Keynes used the weighting function $c=c(p)$. Both weighting functions incorporate non additivity and non-linearity.

This uncertainty about who and/or who is not infected has major ramifications economically. For example, it was the major reason why the restaurant industry has been completely shut down twice in California, even after it appeared that outdoor dining was much safer than indoor dining due to the breezes outside the restaurant breaking up and dispersing the virus into much smaller concentrations. The problem was that when eating, one has to take off one's mask. If people dining outside are near a person who looks healthy, but who is an asymptomatic carrier, then there is a substantial chance that they will contract the virus if they are engaged in conversation for more than 1/2 of an hour with that individual.

This paper seeks to provide results on how college students make a decision on whether to buy or sell gold in order to deal with the financial impacts of the novel corona virus on their lives. Do they use the Neoclassical probability - risk approach, the Tversky-Kahneman-Keynes decision weight-uncertainty approach, or both, in a classroom setting?

Based on the results obtained in our study, we argue that the current reliance on teaching only CAPM and SEU models needs to be broadened so as to deal with cases, such as the Great Recession of 2007-2009 and the Pandemic of 2020-2021, where uncertainty, as well as risk, needs to be taken into account formally.

METHODOLOGY

The methods used in obtaining our results involve the use of different theoretical approaches to decision making under conditions of risk versus uncertainty. The theory discussion is broken up into three component parts. These different approaches are combined with a descriptive statistical approach based on a set of ten questions. Both the Tversky-Kahneman and Keynesian approaches, as opposed to the neoclassical approach, contend that decisionmakers do not use formal, inferential statistical analysis, which combined with hypothesis testing. Instead, they make use of statistics in a heuristic manner.

We obtained our data by using a questionnaire composed of 10 survey questions. Each question was designed to ascertain how undergraduate students used the lectures in the course about decision making to hypothetically reach a decision involving actions concerning their finances and investments, when considering how to act when faced with the possible, negative repercussions resulting from the impacts of the novel corona virus.

THEORY I: NEOCLASSICAL AND MAINSTREAM VIEWS OF DECISION MAKING UNDER RISK

Modern undergraduate training in economics is basically what can be termed "new" Neoclassical economics, where the "new" means that the old neoclassical economics foundation based primarily on the calculus has been updated and re-expressed using various more advanced mathematical techniques, such as the calculus of variations, optimal control theory (Pontryagin's Maximum principle) and dynamic programming (Bellman's equation).

What has not changed, however, since the time of Jeremy Bentham's book, the *Principles of Morals and Legislation*, written in 1787, has been the claim that all decision makers, who are rational, use only the mathematical laws of the calculus of probabilities in making their decisions, which is combined with the principle of maximizing one's utility. This means that the probabilities have to be modeled as exact and precise single numbers that are additive and sum to one. This requires that the decision maker can specify in advance all of the possible different outcomes or options that might occur before he has to make any calculation about what decision to take among those listed. There is no doubt, ambiguity, vagueness, unclearness, or uncertainty involved, only risk attitudes involving the standard deviation, which can be calculated exactly. The same conclusion holds about the shape (curvature) of the decision maker's utility function, which represents the tradeoff between reward and risk. Thus, ALL decisions made by all consumers and producers are modeled as if they were solving the following problem, which requires an expected value analysis. It can be generalized using Savage's Subjective Expected Utility (SEU) approach by using an expected utility analysis by replacing outcomes with the utility of the outcomes:

Suppose you are given an urn with 5 black balls, 7 red balls, 4 green balls, 3 white balls and 6 orange balls. Suppose that you will win \$1.00 if you draw a black ball, \$0.80 if you draw a red ball, \$0.60 if you draw a green ball, \$0.40 if you draw a white ball and \$0.20 if you draw an orange ball. I charge you \$0.75 to play one time or more by drawing from the urn with replacement of the drawn ball. What is the expected value (gain or loss) of playing one time? Should you play or not? Assume linear utility so one need not consider the convexity-concavity of the utility function.

A decision maker can figure out all of the objective probabilities and expected values BEFORE ever playing [$P(B)=.20, P(R)=.28, P(G)=.16, P(W)=.12$ and $P(O)=.24$].

The EV for one draw is

$(.20)(\$1.00)+.28(\$0.80)+.16(\$0.60)+.12(\$0.40)+.24(\$0.20)=.200+.224+.096+.048+.048=.616=.62=62\text{cents}$. The expected loss is $.62-.75=-\$0.13$ loss. So, it is rational NOT to play based on the expected value. On the other hand, if the charge were \$0.50 to play, then it would be rational to play because $.62-.50=+\$0.12$ gain.

The decision maker could also calculate the standard deviation, which is viewed as a measure of risk. He could then take this into account before deciding to play or not to play.

The reigning economic decision theories only uses the probability of success, p , combined with a subjective utility function introduced by Savage, which is a nonlinear function having concave and convex shapes, of the outcomes $U=U(O)$, where O is the outcome. If $U(O)$ is linear, then you

obtain the same result that an expected value analysis, covered above in our urn ball model. The Savage generalization of EV would give the following representation:

Maximize the $SEU = \sum p_i U_i(O)$ over the set of all outcomes being analyzed $i = 1, \dots, n$. In our urn ball model, there were 5 outcomes. An Expected value analysis is equal to $EV = \sum p_i O_i$.

L. J. Savage essentially generalized the concept of probability to include subjective probability, so that it incorporates subjective probabilities, instead of the objective probabilities given in our urn ball example, in his theory of subjective probability. Savage presents his theory of SEU as applying to all types of problems, not just the urn ball kind of problem that we have used for a specific example, although he specifically restricted it to short run and micro applications which he described as “small worlds”.

THEORY II: THE PROSPECT THEORY OF TVERSKY AND KAHNEMAN AND UNCERTAINTY

The Kahneman-Tversky (K-T) Prospect Theory is based on the results of extensive laboratory analysis of statistical decision making which shows that most decision makers do not apply the mathematical theory of probability and/or statistics when making decisions involving statistical outcomes. Instead, they rely primarily on three heuristics, which K-T call the representative heuristic, the availability heuristic and the anchoring heuristic.

The representative heuristic involves basing one’s assessment of probability on the similarity the decision maker perceives or believes exists between one object or event and another object and event. K-T point out one of a number of problems with this heuristic:

“...if probabilities are assessed by representativeness, then the judged probability of a sample statistic will essentially independent of sample size...” (Tversky and Kahneman, 1992, p.6, 1982).

The availability heuristic is applied in situations

“...in which people assess the frequency of a class or the probability of an event by the ease with which the instances or occurrences can be brought to mind.” (Tversky and Kahneman, 1982, p.11; boldface, underline and italics added).

One of the problems here is that

“... a class whose instances are easily retrievable will appear more numerous than a class of equal frequency whose instances are less retrievable.” (Tversky and Kahneman, 1982, p.6).

The third heuristic is called anchoring (or adjustment and anchoring). This heuristic is applied in situations were

“... people make estimates by starting from an initial value that is adjusted to yield the final answer.” (Tversky and Kahneman, 1982, p.14). One of the problems here is that

“... different starting points yield different estimates, which are biased toward the initial values.” (Tversky and Kahneman, 1992, p.14, 1982).

Therefore, even if decision makers use the same kind of evaluation procedure to assess the likelihood of an event or outcome, they will arrive at different answers if their starting point, or initial condition, is different.

Mathematically, the technical analysis used in Kahneman’s and Tversky’s approach results in the specification of nonlinear and non-additive decision weights, π , where the decision weights, π_i , are functions of the probabilities, $p(p_i)$ so that there are nonlinear transformations of the probabilities

The formula that K-T assumes for the evaluation phase is (in its most general and simple form) given by the following formula:

Maximize Expected Weighted Utility, $V = \sum \pi_i(p_i) v_i(O_i)$, where v is a value function that depends on the particular anchor (starting point) chosen to evaluate the outcomes, O , and π is the weighting function that transforms the linear probabilities, p , into nonlinear decision weights, $\pi(p)$.

T-K, who presented much evidence showing that the manner in which decision makers made use of statistical analysis, for example, time series data, to implement the SEU theory used by mainstream economists and decision theorists, is not how a substantial majority of decision makers in a real world of uncertainty, vagueness, ambiguity, and partial incomplete information and data, make decisions, for instance, about whether or not to buy or sell actual gold or gold futures. These kinds of decision are made using some version of Keynes’s approach or in a way that follows from an understanding of how Prospect theory is being applied, which is heuristically based. The mathematics of this approach are, like Keynes’s (see section below), non-additive and nonlinear, which reflects the incompleteness of the data and is not linear and additive, as in SEU theory, which assumes and requires complete data sets. The possible outcomes are interpreted in a manner that is consistent with the application of Prospect Theory by decision makers. π

The actual weighting function, $\pi = \pi(p)$, is given in the 1992 updated version of Prospect Theory, called Cumulative Prospect Theory. It is

$$\pi = \pi(p) = \delta p^\gamma / [\sigma p^\gamma + (1-p)^\gamma], \text{ where } 1-p=q.$$

(see Gonzalez and Wu for the mathematical derivation on pages 139-142).

The crucial teaching point is that π transforms linear and additive probabilities, p and q values, into nonlinear and non-additive decision weights, π . This is identical to what Keynes proposed in 1921 (p.315). It represents the first attempt at modeling a weighting function.

-THEORY III: KEYNES’S A TREATISE ON PROBABILITY APPROACH AND UNCERTAINTY

Consider the following statement made by Keynes in the GT:

" Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as a result of animal spirits — of a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities. Enterprise

only pretends to itself to be mainly actuated by the statements in its own prospectus, however candid and sincere. Only a little more than an expedition to the South Pole, is it based on an exact calculation of benefits to come... We are merely reminding ourselves that human decisions affecting the future, whether personal or political or economic, cannot depend on strict mathematical expectation, since the basis for making such calculations does not exist; and that it is our innate urge to activity which makes the wheels go round, our rational selves choosing between the alternatives as best we are able, calculating where we can, but often falling back for our motive on whim or sentiment or chance." (Keynes, GT, 1936, pp.161-163)

Keynes's consideration of the difficulties of assigning "exact", "strict", "quantitative", "precise probabilities dealing with real world financial and investment decisions might have led Keynes to have made formal use of his conventional coefficient of risk and weight, c , where

$$c = 2pw / (1+q)(1+w),$$

which Keynes had devised in his 1921 *A Treatise on Probability* on p.315. His c coefficient model does not involve exact or strict or quantitative calculations of probabilities. It is not a mathematical expectation but transforms the probabilities/expectations into inexact or approximate measures.

It can be rewritten as $c = p[1/(1+q)][2w/(1+w)]$, where $[1/(1+q)][2w/(1+w)]$ is Keynes's weighting function. Simply reversing the numerator and denominator of Keynes's weighting function allows a decision maker to evaluate overweighting of probabilities instead of the usual underweighting problems considered in this paper. Keynes's formula represents an imprecise approach to probability and expectations.

Let O be our outcome. Then we seek to

Maximize $cO = p[1/(1+q)][2w/(1+w)]O$ instead of pO .

It is easy to substitute $U(O)$ for O to obtain an expected utility analysis, which is weighted by the factors $[2w/(1+w)]$ and $[1/(1+q)]$.

Keynes's basic decision weight is $p[2w/(1+w)]$. This is basically transforming linear p 's into nonlinear c 's to consider the limited availability of data or knowledge or evidence about the different options. It is now easy to see that the KT weighting function, $\pi(p)$, plays a role in Cumulative Prospect Theory that is equivalent in their system to Keynes's weighting function, $c(p)$, in his decision theory.

Keynes also incorporates an additional second weighting factor, $[1/(1+q)]$, to account for nonlinear risk probability preferences, as opposed to linear preferences, into his c coefficient.

Keynes does not have an explicit value function, V .

However, both the T-K $\pi(p)$ function and Keynes's $c(p)$ function is doing the same thing—transforming linear probabilities into nonlinear decision weights due to perceptions of incomplete data or information on the part of decision makers. What is leading decision makers to incorporate π or c is a perception of data/information/evidence/knowledge incompleteness or insufficiency. This means that probabilities

are viewed as being uncertain, imprecise or inexact and not the precise, exact, single, linear and additive probabilities of mainstream economics approaches used in the SEU and CAPM models.

Based on these different theoretical approaches, two charts were constructed that are descriptive in nature. The charts show visually how the students reacted to the concepts of uncertainty (not able to accurately estimate the risk due to deficiencies in the data available to them at the time they needed to make a decision) and risk (able to accurately calculate relevant outcomes based on complete data sets). The charts show that the students were thinking in terms of both concepts, risk and uncertainty, as being applicable in how they would go about making a decision if actually faced with the need to make a decision. (See pages 15 and 16 below).

RESULTS:

The following questionnaire was asked of 128 students, of which 124 provided answers:

1. Do you invest in the stock market?
2. Do you have more confidence in Gold as an investment option?
3. Has COVID affected your decision in stocks/gold investments?
4. Has COVID made you more uncertain about the stock market?
5. Does uncertainty play a factor in your investments?
6. Would you buy Gold as an investment?
7. Do you have confidence in the stock market?
8. If you invest, do interest rates affect your stock market decisions?
9. If you invest in Gold, do interest rates affect your Gold Investing decisions?
10. Does risk vs reward affect your investment decisions?

The students were given the following five choices for each question:

- a. Definitely Not
- b. Not Likely
- c. Neutral
- d. Likely
- e. Definitely

These questions allowed us to determine how important the roles of risk (the mainstream neoclassical approach) or uncertainty (Kahneman-Tversky and Keynes approaches) were in the student's decision-making approach.

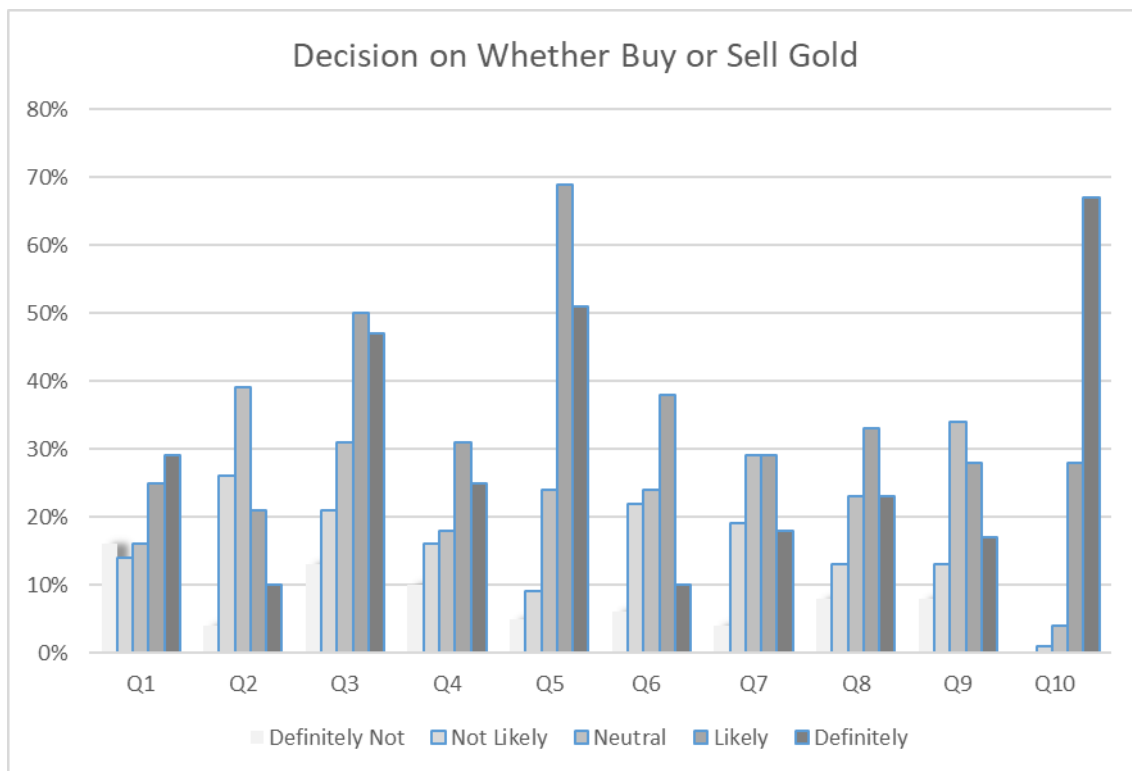
The following summary shows the percentages of a) to e) answers to each of the 10 questions from the 124 students.

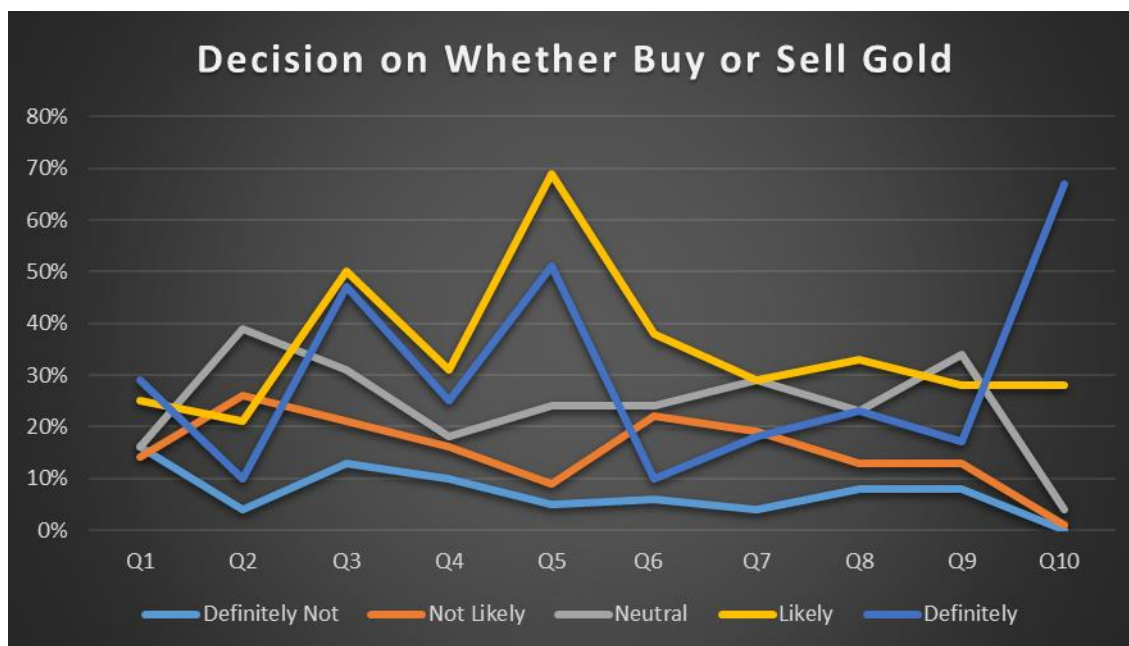
The student population was made up of Asian, 10.5%, African American, 7.1%, Hispanic, 29.3%, Hawaiian, 0.7%, White, 25.1%, and Unknown Race, 1.1%. The mix of male and female students in each class was about 55% to 45%.

The average age of the students was 35-45. The great majority, 80 %, of the students, were middle class. Students in two sections of the Business Analytics IS335 course, which was

an online, night class, served as our data source. Students were given 15 minutes each to answer the questions:

Survey Questions	Choices				
	Definitely Not	Not Likely	Neutral	Likely	Definitely
	Survey Results				
Q1: Do you invest in the stock market	16%	14%	16%	25%	29%
Q2: Do you have more confidence in Gold as an investment?	4%	26%	39%	21%	10%
Q3: Has COVID affected your decision in stocks/gold investment?	13%	21%	31%	50%	47%
Q4: Has COVID made you more uncertain about the stock market?	10%	16%	18%	31%	25%
Q5: Does uncertainty play a factor in your investments?	5%	9%	24%	69%	51%
Q6: Would you buy Gold as an investment?	6%	22%	24%	38%	10%
Q7: Do you have confidence in the stock market?	4%	19%	29%	29%	18%
Q8: If you invest, does interest rates affect your stock market decisions?	8%	13%	23%	33%	23%
Q9: If you invest in Gold, does interest rates affect your Gold Investing decisions?	8%	13%	34%	28%	17%
Q10: Does risk vs reward affect your investment decisions?	0%	1%	4%	28%	67%





The two main neoclassical models, which use either the standard deviation as a measure of risk (CAPM-Capital Asset Pricing Model) or the shape (the curvature of) of the utility function as a measure of risk (SEU-Subjective Expected Utility), were measured by question 10. The students were definitely using the risk concept put forward in the neoclassical, mainstream approaches that concentrate on a tradeoff between risk and reward.

However, the mainstream approach denies that questions of doubt, uncertainty or confidence should play any role in decision making because the linear, additive probability estimate, which is the mean of a normal probability distribution in the CAPM model or p in the SEU model, is claimed to represent confidence, uncertainty or doubt. Both the K-T and Keynes approaches argue that uncertainty, which will have a nonlinear impact, represents doubt about the amount of confidence a decision maker has in his probability estimate, where the K-T nonlinear weighting function, π , is a function, $\pi = \pi(p)$, that transforms linear probabilities into nonlinear decision weights, π . This is very similar to Keynes's c , his conventional coefficient, which also transforms linear probabilities into nonlinear decision weights (conventional coefficients), where $c = c(p)$.

Keynes's w variable, discussed in the section on Keynes above, can also be viewed as a separate measure of confidence, where confidence would be a positive function of w , where $0 \leq w \leq 1$, and w is Keynes's measure of the completeness of the available information to the decision maker. The available information was one of the three heuristics that make up the K-T Prospect theory.

However, the answers given by the students in Questions 2, 4, 5, and 7 show that they are also considering uncertainty (doubt) and confidence as being separate, relevant variables. The students were concerned that, in reaching a decision, they needed, in addition to the mainstream model using only linear and additive probabilities, a model incorporating nonlinear and non-additive decision weights.

We interpret these results as leading to the conclusion that all three theories of decision making are relevant and should be taught. However, this is not the case. Prospect theory is generally taught only in Psychology and Philosophy courses, while Keynes's approach is only taught in Philosophy courses under the title of imprecise probability. Economics Departments only teach the CAPM and SEU approaches, based on additive p values, and do not teach about the need to use non additive decision weights.

The choice of a large sample size (30 or more observations from a population) was made so as to reduce the risk that the sample that was used was not representative of the general population of decision makers as a whole.

The paper used descriptive statistics, as opposed to inferential statistics, as the goal was not to provide a test of a specific hypothesis regarding questions about which model of decision making (Neoclassical, Tversky-Kahneman, Keynesian) makes the most accurate predictions, but, given the evidence provided by Tversky-Kahneman, that decision makers are not using inferential statistics, how do they go about making decisions that are based only on immediately available evidence, which ignores evidence from the past, which the decision maker is not taking into account. Decisions are being made using only part of the total evidence. Keynes would describe this decision situation as being one of low Evidential weight of the argument Decision making under such circumstances needs to be explicitly recognized theoretically. The use of either, or both, the Tversky-Kahneman and Keynes approaches to transforming probabilities into decision weights makes decision theory more robust.

The problem of considering an approach to decision making using partial evidence was considered by L. J. Savage under the name, vagueness. As is discussed in the conclusion regarding the deficiencies in the paper, Savage argued that vagueness would substantially complicate the decision-making process, resulting in hesitancy on the part of decision makers to base their decisions on subjective probabilities.

However, Savage did not believe that there was any way to satisfactorily deal with the problem of vague evidence technically. This may be the reason that Savage restricted the applicability of his theory to "small" worlds alone, which involved short run periods of time at the micro level and excluded long run, intertemporal decision making, and macro considerations. (See Savage 1954, p. 16-17).

CONCLUSIONS

Based on an analysis of the student answers to our questionnaire, we conclude that the decision to buy/sell gold or gold futures was heavily impacted by the uncertainty (doubt) created by the appearance of the novel corona virus, as well as by a lack of confidence in the capability of the standard, neoclassical problem specification approach to deal with the Novel corona virus. Student answer were not only a function of perceived, precise, subjective probabilities and rewards (utilities), but involved a belief that the data itself was uncertain and ambiguous, leading to a lack of confidence in other asset choices available besides gold or other hard assets.

Gold has been regarded as an optimal choice for many centuries when questions arise about the uncertainty/doubt of the future. This concern with uncertainty and/or confidence and choice of gold, entails far more than just a risk/return(reward) tradeoff based on the standard deviation or curvature of a decision maker's utility function. When there is a small degree of doubt/uncertainty/lack of confidence, then the neoclassical model should be emphasized, while if there is a substantial amount of doubt/uncertainty/lack of confidence, then the Tversky-Kahneman/Keynes approaches should be emphasized.

We would recommend that economics departments cover both the Kahneman-Tversky and Keynes models as complements and supplements to their CAPM and SEU models in their undergraduate courses in microeconomics. In our view, students would then have more tools (models) in their toolbox or repertoire to choose from when facing complex decision environments.

A deficiency in our study is how to actually measure with precision the impact of uncertainty on decision makers. Savage's concern about the impact of vagueness on decision makers, despite his inability to come up with a precise method of how serious the impact on decision makers was regarding the differences in risk assessment, as opposed to uncertainty (vagueness) assessment, can be incorporated by using the formulations of Tversky-Kahneman or Keynes.

Our data is qualitative; it shows that decision makers do consider the differences between acting under risk and/or uncertainty (vagueness). However, it does not provide precise, quantitative measures of which situation, decision making under risk or decision making under uncertainty, is regarded as the most prevalent and/or most important to decision makers. All that we can say from this study is that decision makers are aware of the differences and are using both

approaches. The particular mix of techniques chosen by students was not studied.

Savage (see Savage, 1971, pp.597-598) argued that the only way to correct for vagueness (uncertainty) in the data, being used by decision makers to calculate their subjective, was to teach decision makers better and more accurate elicitation techniques as to what their subjective probability assessments and estimates were, as well as their estimation of the utilities of the outcomes. Savage argued that Proper scoring rules needed to be applied so that different types of estimation techniques would have to be actually compared and judged as being either better or worse than the alternative approaches available instead of it just being assumed that one approach was superior based on mathematical elegance, beauty and tractability.

CONFLICT OF INTEREST

The authors reported no potential conflict of interest.

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