Asset allocation by Unsupervised Learning

Ismail Lotfi[1,2,*1], Lamiae Megzari[3,2] and Abdelhamid EL Bouhadi[1,3]

[1]Laboratoire de Recherche et d'Etudes en Management, Entrepreneuriat et Finance (LAREMEF), ENCGF, Sidi Mohamed Ben Abdellah University, Fez, Morocco.

[2]Laboratoire Systèmes et Environnements Durables (LSED), Faculté des Sciences de l'Ingénieur, Université Privée de Fès, Fez, Morocco.

[3]Laboratoire d'Etudes et Recherche en Management des Organisations et des Territoires (ERMOT), FSJES, Sidi Mohamed Ben Abdellah University, Fez, Morocco

Abstract: Modern portfolio theory is closely linked to the concept of diversification. As a result, the most important decision of investor is to make his allocation asset portfolio more and more efficient. Thus, for a given level of risk, the investor seeks to maximise the expected return and minimise the risk by constructing an optimal portfolio. In this paper, we seek to know how unsupervised learning can be used to define the asset allocation strategy. In this sense, we have carried out a comparative study between a so-called classical portfolio, which is based on the modern portfolio theory (i.e. a portfolio constructed on the basis of numerical optimisation) and a portfolio based on unsupervised learning. The aim of this comparison is to look for the best performing method that can give the best asset allocation. Our findings show that the optimal strategy for an ambitious investor lies to the unsupervised learning algorithms that allow a dynamic analysis of portfolio. However, the optimal strategy for a risk-averse investor is still the numerical optimisation approach.

Keywords: Independent Component Analysis, Efficient Frontier, Asset Allocation, Optimisation, Dynamic Simulation.

1. INTRODUCTION

Controlling the risks associated with market activities has become a major concern for both financial institutions and investors. Choosing a best portfolio is resulting from a trade-off between the risk incurred by the investor and the expected return. For a given level of risk, the investor seeks to maximise the expected return by constructing an optimal portfolio assets, which are combined with a loan or borrowing in order to obtain the level of risk the investor wishes to bear (Varian 1993).

Thus, asset allocation is the most important decision that any investor faces to construct a portfolio, as there is no single solution that will be suitable for all investors. The decision making is mostly linked to risky aversion. Closely related in diversification portfolio modern theory (Sharpe 1964), the basis analysis is starting simply with linear mean-variance framework. This theory assumes that a manager composes his portfolio with different types of assets (Markowitz 1959). The main objective of this diversification is to reduce the risk incurred by the investor while maintaining a satisfactory level of return, since there is a low probability that all the investments (the different assets that make up the portfolio) will decrease in return at the same time. Statistically, the concept of diversification is represented by the correlation matrix between assets. Basically, the lower of level of correlation between assets, the better the diversification it is.

In this article, we are looking for the best asset allocation that will allow us to build an optimal portfolio. This simply means that try to maximise return while to minimise risk through optimal diversification. To this end, we will show the contribution of Machine Learning algorithms in the portfolio construction by comparing it with a standard variance-covariance analysis.

2. ASSET ALLOCATION

2.1. Modern Portfolio Theory

In 1952, Markowitz spearheaded the portfolio management model usually used by many financial institutions. Markowitz believes that economic agents are inherently risk averse,

*Address correspondence to this author at Laboratoire de Recherche et d'Etudes en Management, Entrepreneuriat et Finance (LAREMEF), ENCGF, Sidi Mohamed Ben Abdellah University, Fez, Morocco;
E-mail: lotfiismail09@gmail.com

[1] https://orcid.org/0000-0003-1277-6271
[2] https://orcid.org/0000-0002-9988-6547
[3] https://orcid.org/0000-0003-3120-2628

and distinguishes them through a quadratic utility functions. In their choice, agents rely solely on two main elements in the random distribution of their wealth, namely expectation and variance (i.e. risk-return trade-off). Indeed, the expected utility of an individual's wealth would be a function of mean and variance portfolio profitability. Consequently, Markowitz was able to formulate the portfolio problem by establishing the central theorem of Mean-Variance (Return-Risk) approach. This theorem consists to maximise the expected portfolio return for a given variance or minimising the variance (risk) for an expected target return. These two principles lead to formulate efficient frontier, which the investor can choose his preferred portfolio that perfectly matched with his risky preferences (Markowitz 1959).

A. Efficient Frontier by Monte Carlo Simulations

By defining combinations of securities according to their rate of return and risk level, the most optimal portfolios in the market are those that offer the highest expected return for different levels of risk assumed, or alternatively, those that offer the lowest risk for any targeted return. Thus, the graphical representation takes the form of a curve in which any portfolio below the cut-off line is considered inefficient. The owner of such a portfolio will, therefore, be exposed to unnecessary risk, or will receive a lower return than he could possibly obtain given the risk taken.

To this end, the relationship between risk (which is usually represented in finance by volatility and measured by standard deviation) and return is graphically represented by a curve that clearly shows the limits of an efficient portfolio. This curve is also called the "Markowitz frontier" and serves as a reference for rational decision-making in investment solutions.

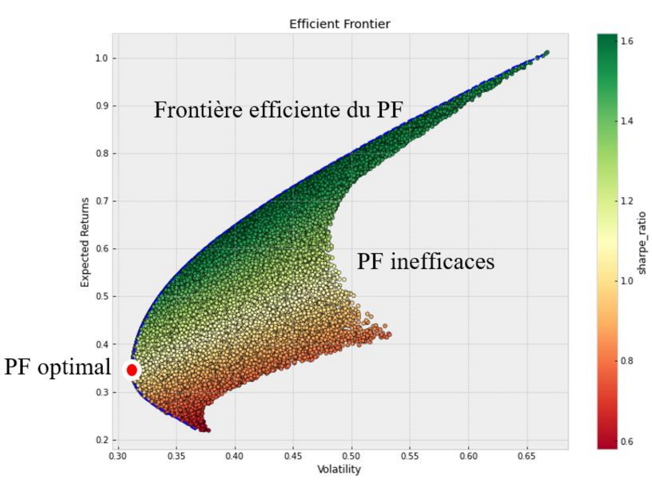


Fig. (1). Markowitz efficient frontier.

As shown in the graph above, all portfolios below the efficient frontier are considered inefficient. It will be therefore recommended to retain a portfolio that lies to the frontier. Consequently, the allocations retained are those that result in the optimal portfolio and that maximise the Sharpe ratio and/or minimise the risk (which is reflected in the volatility of portfolio). Mathematically, this trivially translates into an optimisation problem under a constraint. Hence, the solution of the problem is as follows:

$$\begin{cases} Max\, R_{PF} = \Sigma_1^k \omega_i R_i \\ \text{and} \\ Min\, \sigma_{R_{PF}} \end{cases} \qquad (1)$$

Where:

- $\boldsymbol{R_{PF}}$: Portfolio return
- $\boldsymbol{R_i}$: Assets return
- $\boldsymbol{\omega_i}$: Weight of the asset in the portfolio
- $\boldsymbol{\sigma_{R_{PF}}}$: Standard deviation of the portfolio

B. Efficient Frontier by Optimisation

The efficient frontier mentioned above is a rudimentary approach based on Monte Carlo simulation. It allows us to obtain a set of portfolios that provide the expected return for a given level of risk. Based on this principle, we can use numerical optimisation to minimise the volatility the portfolio. Mathematically, the problem is describe as follows:

$$\begin{cases} Min\, \omega^T\, \Omega\, \omega \\ \omega^T\, R_{PF} = r_p \\ \Sigma\, \omega_i = 1 \;\; et\; \omega_i \geq 0 \end{cases} \qquad (2)$$

Where:

- $\boldsymbol{\omega}$: Vector of asset weights
- $\boldsymbol{\Omega}$: Variance-covariance matrix
- $\boldsymbol{r_p}$: Expected return of portfolio

This risk minimisation problem can be inverted to transform the expectation-variance optimisation problem back into a risk-adjusted return maximisation:

$$\begin{cases} Max\, \omega^T\, R_{PF} - \gamma\, \omega^T\, \Omega\, \omega \\ \Sigma\, \omega_i = 1 \;\; et\; \omega_i \geq 0 \end{cases} \qquad (3)$$

Where γ is the risk aversion parameter $(\gamma \in [0, \infty[\,)$

2.2. Factorial Analysis

A. Principal Component Analysis

When a large number of quantitative variables are studied simultaneously (this is the case of a portfolio composed of several assets), the difficulty is linked to detect the multicolinearity. And, by extension, the best choice of the weights of the assets that will constitute our portfolio in order to satisfy the principle of diversification. Hence, we consider the importance of Principal Component Analysis (PCA) to solve the optimisation problem. The main objective of this analysis is to reduce the information held by highly correlated variables by creating dummy variables while distorting reality as little as possible (Billette 2013).

So, in order to synthesise the quantitative data table, the principal component analysis (PCA) can be given by a reduced number of dummy variables F^1, F^2,…, F^m, by a kind of linear combination of centred and reduced variables

$Z_1, Z_2, \ldots, z_p$. In the other words, PCA proceeds by linear transformations of a large number of inter-correlated variables in order to obtain a relatively limited number of uncorrelated components[4] (Yang 2015).

This approach has a crucial interest, as it facilitates analysis by grouping large data into smaller sets to eliminate problems of multicolinearity between variables. This procedure can only be done by variance-covariance matrix or correlations helping (which depending on the type of PCA).

In this sense, PCA aims to represent, in graphically form, the essential information in a table of quantitative data consisting of *p* variables and *n* observations.

B. Independent Component Analysis

Independent Component Analysis (ICA) is a recently developed factor analysis method that extends Principal Component Analysis (PCA). This method allows a linear transformation of a set of variables assumed be non-Gaussian[5] in order to determine "independent" components. Independent Component Analysis (ICA) is similar to Principal Component Analysis (Cherfi et al. 2007). While PCA only takes into account a zero correlation between the principal component, which is not sufficient to affirm the independence of the axes, ICA fills this gap by making it possible to obtain not only uncorrelated but also independent components (Fokou 2006).

Based on this observation, we will use the ICA instead of the PCA to calculate the factorial axes, while ensuring their independence. This will allow us to obtain the financial asset allocations of a portfolio in the form of a linear combination of an independent factorial axis. This assumption can then, satisfy the principle of diversification (Lei 2019).

3. METHODOLOGY

The objective of this paper is to determine how unsupervised learning[6] can be used to define the asset allocation strategy in portfolio construction. This process involves information mining, structure identification and determination of data correlations and does not require the existence of a variable to be explained, as in the case of supervised learning.

Unsupervised learning will be used in our study through two levels:

- Differentiation of risk factors by applying AIT to asset returns: this is an approach that uses factor analysis to extract information from the data and estimates the influence of the data on returns;
- Building a portfolio based on the main components of the ICA.

The interest of such reflection is to compare the performance of a portfolio created through an artificial intelligence algorithm against a portfolio based on modern portfolio theory (Snow 2020). Thus, the methodology used will consist, first, in determining the best assets likely to make up our portfolio. This step consists in reducing the number of assets by making it more limited. Therefore, the choice will be made on the basis of stocks that simultaneously have a high potential return (measured by EPS[7]) and low volatility (measured by the standard deviation). In the next step, we will try to identify the data generating process and links between series.

To this end, we will use factor analysis, more precisely the PCA. The objective is to reduce the dimensions in the identification of data structures. Then, in order to successfully construct our portfolio, we will extract the weights of the different assets, due to an independent factor analysis (ICA). Finally, we will compare the performance of this portfolio generated by the ICA with that of a second portfolio generated by modern portfolio theory, (i.e. by Markowitz efficient frontier).

The assumptions made in this work are based on four main points:

- The source variables are statistically independent,
- Linear transformations are sufficient to capture the information,
- The factor components do not follow a normal distribution,
- The correlation matrix *R* is invertible.

These assumptions, usually tested in finance, are necessary for the application of an effective independent factor analysis (ICA).

3.1. The Data

Following the approach proposed in the methodology above, we will carry out a comparative study between a so-called standard portfolio construction, which is based on modern portfolio theory, and a portfolio based on unsupervised learning. The aim of this comparison is to know the most efficient method that can give us the best composition of portfolio.

For this purpose, we tried to identify the number of stocks based on two criteria: high return (measured by EPS) and low stock volatility. These criteria allowed us to select 7 stocks, namely:

- *NEJ*: the company "Auto Nejma" which imports and markets vehicles;
- *ALM*: the company "Aluminium du Maroc S.A." specialising in the manufacture of aluminium;
- *PRO*: the company "Promopharm" specialised in the production and selling of pharmaceutical products;
- *SOT*: the "Sothema" company whose activity is the manufacture of medicines;
- *CTM*: the "CTM" company which specialising in road transport;
- *ATW*: Attijariwafa Bank is the biggest banking group in Morocco;

[4] This is a very useful technique in Machine Learning to improve the quality of models.
[5] Non-normality is a characteristic of financial series.
[6] Unsupervised learning is a field of Machine Learning.
[7] Earnings Per Share

- *IAM*: “Maroc telecom” is a company specialising in telecommunications.

For the realization of this study, the historical data of stock prices, for the period going from January 2015 to June 2021, were collected from the Casablanca Stock Exchange. This period covering more than 6 years of daily data allowed us to collect a total of 3411 observations. The daily returns[8] are noted successively [r_{NEJ}], [r_{ALM}], [r_{PRO}], [r_{SOT}], [r_{CTM}], [r_{ATW}] and [r_{IAM}].

3.2. The Performance Portfolio Evaluation

When we talk about portfolio performance evaluation, we are simply referring to the endorsement of a procedure that may lead to adjust different portfolios. Indeed, in order to choose the funds (portfolios) in which to invest, the manager must be able to compare them on the basis of one or more criteria. Hence the need for the principle of portfolio performance evaluation, which relies essentially on the calculation of ratios to judge the quality of performance achieved by a portfolio. In practice, there are several performance measurement ratios and methods, including the following:

- **Sharpe ratio**: The Sharpe ratio is probably one of the most famous indicators. The purpose of this ratio is very simple. It measures the profitability of an investment compared to a risk-free investment. The portfolio is given to outperform a risk-free investment if the Sharpe ratio is greater than unity. However, this ratio has several drawbacks, especially in the case of non-Gaussian returns (as is the case of financial series).
- **Sortino Ratio**: The Sortino ratio follows the same framework as the Sharpe ratio, except that it provides a solution to the problem of asymmetric distribution of asset returns. This ratio is mainly based on the standard deviation of excess negative returns instead of the total standard deviation. The particularity of this indicator is that it allows the selection of financial assets for investors who are more interested in downside risk. To this end, a high Sortino indicator will highlight assets that have performed well but are resilient during periods of market decline. Indeed, a ratio greater than 1 indicates that the performance of the portfolio is greater than the incurred risk and demonstrates the efficiency of investor’s management.
- **Omega ratio**: This ratio does not require any assumptions about the distribution of asset returns. Like Sortino’s ratio, it is based on the minimum break-even point instead of the risk-free rate. Unlike Sharpe, the Omega ratio offers the possibility of ranking portfolios, since the values calculated are always positive. If the ratio is greater than 1, then the portfolio will have a positive performance. Conversely, it will have a negative performance.
- **Max drawdown**: This indicator reflects the maximum loss of an asset or strategy during the period under study by comparing it with its maximum profitability. In other words, it measures the loss incurred by an investor in the case where he bought his portfolio at its highest price and sold it at its lowest price.
- **Calmar Ratio**: This ratio aims to compare the annualised return with the max drawdown since the fund creation. It is an index that tries to fill the gap left by other methods. These methods allow for a comparison of a fund that has survived several crises with a newly created fund in a bull market. Therefore, the comparison may be biased by the length of the history and market conditions, which is not obvious. However, although this ratio focuses on drawdown as an indicator of risk, it is less useful statistically. This is due to the neglect of the overall portfolio volatility.
- **Value-at-Risk**: the VaR is used to determine the maximum potential loss that an investor can incur on his portfolio, over a given holding period, under normal market conditions and for a well-defined confidence level.

4. EXPERIMENTAL RESULTS

The most important decision an investor has to make the asset allocation of his portfolio. Indeed, this decision depends on the choice of assets to be included in the investor’s portfolio under risk aversion, given the pursued strategy and investment horizon, etc. The variation of all these factors leads to the absence of a single solution adapted to all investors.

In the present work, we will construct 7 portfolios according to 4 different approaches (basic allocation, simulation, optimisation and factor analysis). Then, we will evaluate the performance of these portfolios in order to know the optimal approach in terms of profitability and risk.

4.1. Building Portfolios

A. Naïve Approach

In this approach, we will create an equally weighted portfolio, i.e. each stock will be assigned an equal weight to all stocks. This is a simple and naive approach, but it can lead to a non-negligible performance as shown by DeMiguel in 2007 (see also, DeMiguel, Garlappi, and Uppal 2009). Indeed, our portfolio noted [PF_{eq}] will correspond to the following linear combination:

$$PF_{eq} = 0.1429 r_{NEJ} + 0.1429 r_{ALM} + 0.1429 r_{PRO} + 0.1429 r_{SOT} + 0.1429 r_{CTM} + 0.1429 r_{ATW} + 0.1429 r_{IAM} \quad (4)$$

B. Markowitz Approach

In a case similar to our study, modern portfolio theory is often used, based on the notion of the efficient frontier. The

[8] Log-return compounded equal to $\log(1+R_t)$ and where $R_t = (p_t - p_{t-1}) / p_{t-1}$ is the simple return.

latter is constructed by a Monte Carlo simulation which based on a set of optimal portfolios.

At this stage, we use the programming language “Python” to perform the Monte Carlo simulation in order to construct 160,000 portfolios with random weights. The visualization of this simulation is shown in Fig. (**2**) as follows:

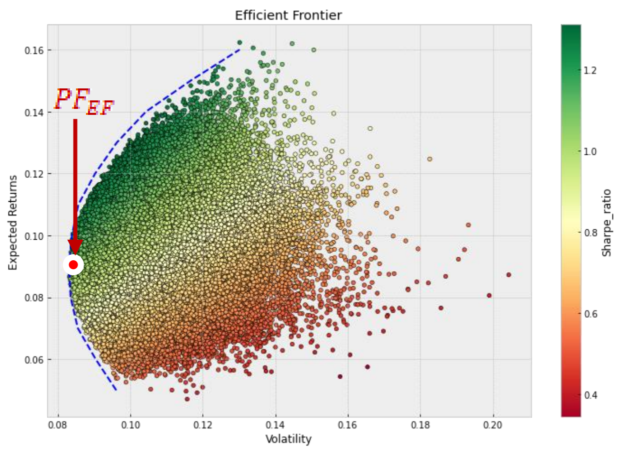


Fig. (2). Efficient frontier by simulation

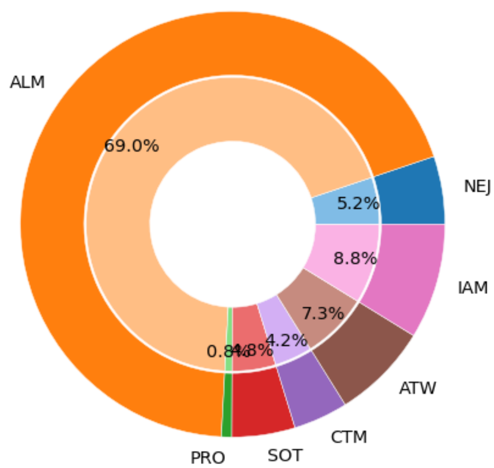


Fig. (3). Markowitz Asset Allocation.

The shape of the frontier is a little irregular because of the simulated values, which are not so frequent in certain extreme zones. The selected portfolio noted [PF_{EF}] is the one that maximises the return and minimises the risk. In this case, we speak about a tangency portfolio. The composition of this portfolio is given in Fig. (**3**). In fact, our portfolio is written as follows:

$$PF_{EF} = 0.052 r_{NEJ} + 0.69 r_{ALM} + 0.0079 r_{PRO} + 0.48 r_{SOT} + 0.042 r_{CTM} + 0.073 r_{ATW} + 0.088 r_{IAM} \quad (5)$$

Optimisation Approach

In this approach, we will use numerical optimisation to find the portfolio that optimises the objective function as shown above. We will use the same assets in the construction of our portfolio noted [PF_{opt}]. Then, we proceed with an examination of this portfolio to get the optimal composition and maintain the most efficient one. The efficient frontier obtained by a numerical optimisation is presented in Fig. (**4**):

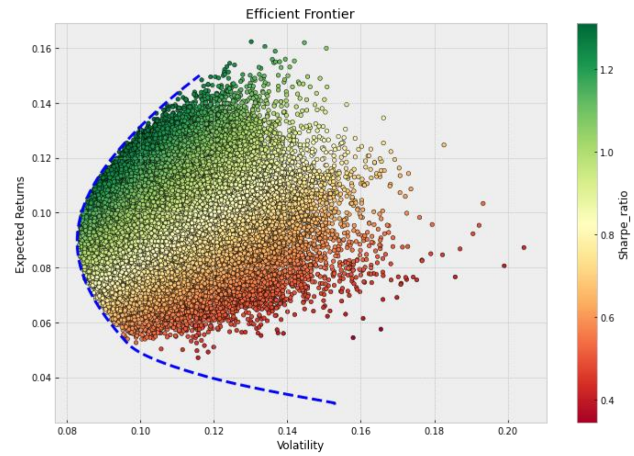


Fig. (2). Efficient frontier by optimization.

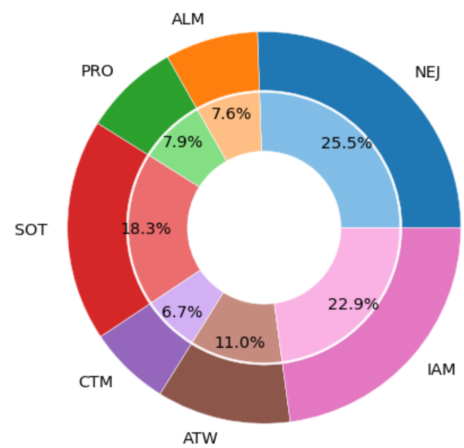


Fig. (5). Asset allocation by optimization.

The identification of the minimum volatility portfolio allowed us to know the asset allocation as presented in Fig. (**5**). As a result, our portfolio is written as follow:

$$PF_{opt} = 0.255 r_{NEJ} + 0.076 r_{ALM} + 0.079 r_{PRO} + 0.183 r_{SOT} + 0.067 r_{CTM} + 0.11 r_{ATW} + 0.229 r_{IAM} \quad (6)$$

The asset allocation in this portfolio is different from that given by the Monte Carlo simulation. In this case, the allocation is mainly distributed between 4 stocks (NEJ, SOT, IAM and ATW), in the opposite of simulation which allocated 69% of the funds to a single stock (ALM).

So far, we have proceeded to optimise the portfolio by minimising its volatility. However, it is also possible to invert the problem towards a maximisation of the risk-adjusted return. In this case, a convex optimisation can be used to find the efficient frontier.

The asset allocation can be obtained by maximising the return depends on the level of risk tolerated by the portfolio manager. Based on this, we will have an optimal portfolio for each risk level. Thus, in Fig. (**6**), we observe the asset allocation as a function of risk aversion:

We deduce that the more risk averse of investor, the more diversified the portfolio. Starting with the stock “SOT” (pharmaceutical company), an investor with low risk aver-

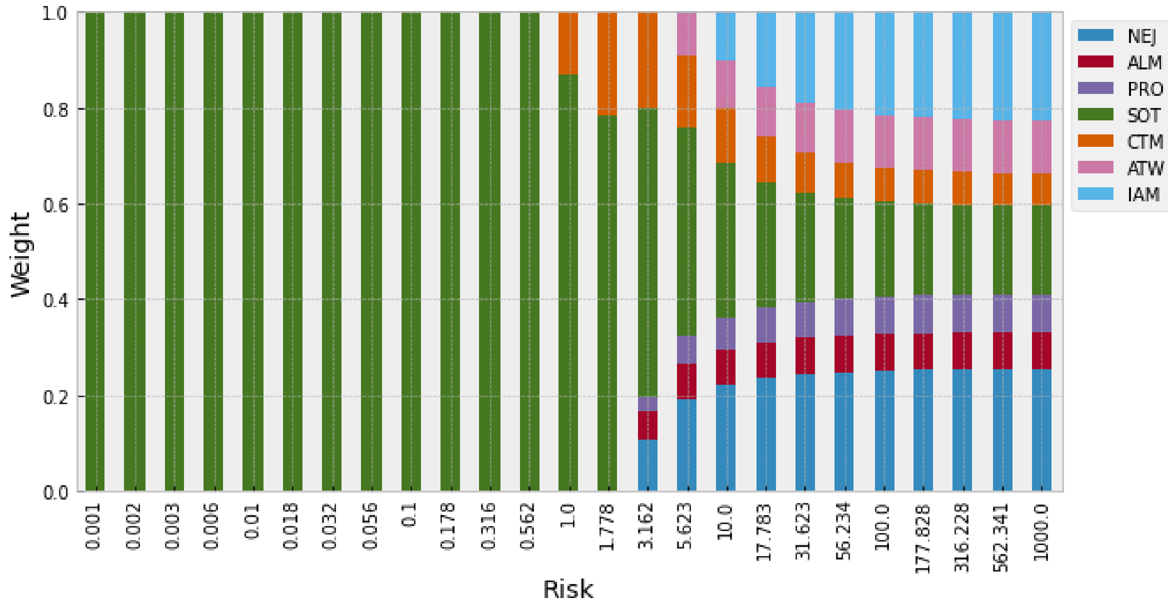


Fig. (6). Risk-based asset allocation.

Table 1. Selected Components.

	NEJ	ALM	PRO	SOT	CTM	ATW	IAM
Portfolio 1	0.005	-0.404	0.868	-0.032	-0.279	-0.064	-0.032
Portfolio 2	0.013	0.833	0.218	-0.084	-0.476	-0.118	-0.105
Portfolio 3	-0.179	0.320	0.394	-0.158	0.814	-0.115	-0.098
Portfolio 4	-0.058	-0.007	-0.011	0.794	0.038	-0.498	-0.341

Source: Author's calculation.

sion could invest all the funds in this stock since it is currently experiencing a strong, unprecedented trend due to Covid19. However, this trend could be reversed abruptly with significant losses.

Indeed, a very risk-averse investor will naturally choose to diversify his portfolio perfectly in order to minimise losses in the event of an unexpected fall by the manager. The allocation to each asset will then be increasingly important according to the risk aversion.

D. Factorial Analysis

In this section, we will use factor analysis to calculate the factorial axes, while ensuring their independence. Based on this, we retain the four most important components. This will allow us to obtain the weights of the assets in a portfolio as a linear combination. The four independent components are presented in the table below (table **1**):

The factorial analysis also allows for a dynamic analysis of the portfolio in that it tells the investor which position to take (James and al. 2019). Furthermore, we distinguish 2 positions:

- The long position which corresponds to a buyer's position. This position is characterised by a positive coefficient in table **1**,
- The short position which corresponds to a seller's position. It is characterised by a negative coefficient in table **1**,

Subsequently, we will determine the weight of each action by dividing the positive (resp. negative) coefficients by the sum of the positive (resp. negative) coefficients in order to normalise the coefficients. This will result in a set of weights in which the sum is equal to 1 (resp. -1). The normalised weights are shown in Fig. (**7**):

The investor will then be able to buy or sell a stock depending on the evolution of the financial market and/or the arrival of new information. Consequently, we obtain 4 portfolios, noted respectively [PF_1], [PF_2], [PF_3] and [PF_4], from 4 standardised components.

4.2. Benchmarking of Allocation Strategies

After having constructed several portfolios according to different strategies, we will discuss their performance against several criteria. All the criteria used in this evaluation are calculated using Python 3.8. The results of our analysis are presented in the table **2**:

The comparison of different strategies shows that factor analysis is considered to be the best performing in terms of annual or cumulative profitability. In this sense, we note that

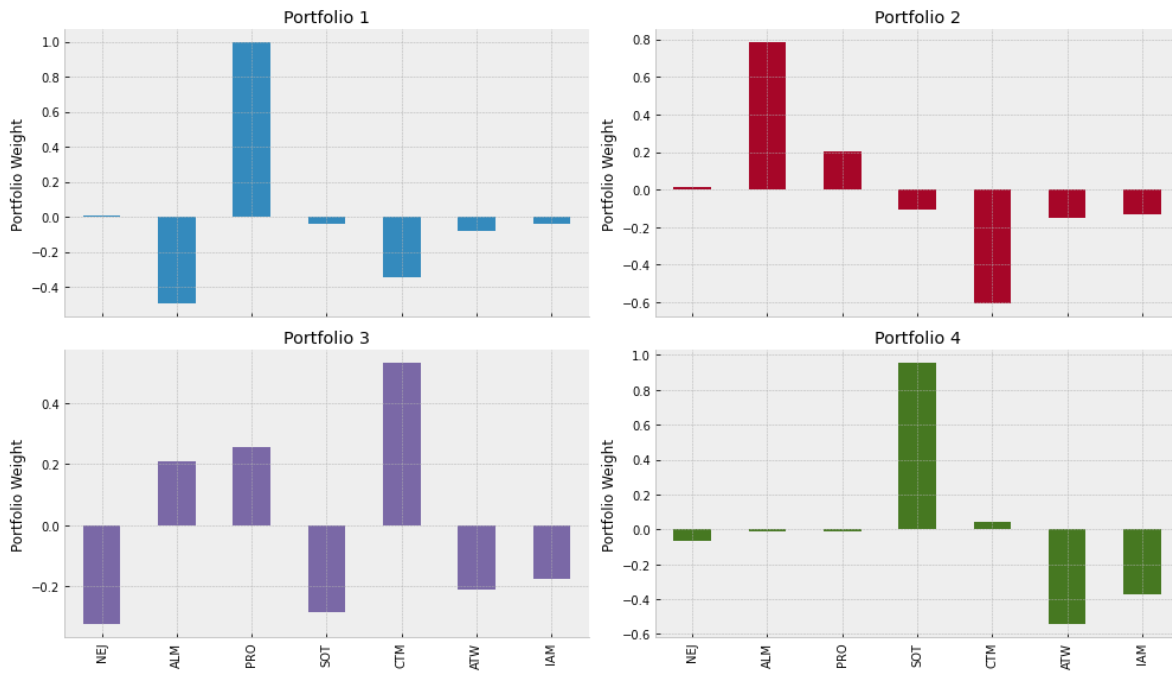


Fig. (3). Asset allocation using factor analysis.

Table 2. Evaluation of the Performance of Portfolios.

	PF_{eq}	PF_{EF}	PF_{opt}	PF_1	PF_2	PF_3	PF_4
Annual return	9.5 %	6.9 %	9.0 %	27.7 %	-10.8%	17.7 %	24.8 %
Cumulative returns	76.9 %	51.7 %	71.5 %	348.0 %	-50.4 %	171.3 %	289.0 %
Annual volatility	9.3 %	20.4 %	8.3 %	4446 %	53.7 %	1659 %	28.2 %
Sharpe ratio	1.03	0.43	1.08	-0.22	0.06	-0.09	0.92
Calmar ratio	0.48	0.18	0.66	0.03	-0.15	0.16	0.82
Omega ratio	1.21	1.13	1.23	0.69	1.01	0.91	1.37
Sortino ratio	1.52	0.62	1.58	-0.25	0.08	-0.09	1.44
Max drawdown	-19.7 %	-38.4 %	-13.5 %	-1074 %	-72.4 %	-110.5%	-30.1 %
Daily value at risk	-1.1 %	-2.5 %	-1.0 %	-564.1%	-6.8 %	-209.6%	-3.5 %

Source: Author's calculation.

[PF_4] achieves an annual return of about 25% (we exclude [PF_1] because of its aberrant volatility). This portfolio also presents the best values for the Calmar and Omega ratios with values of 0.81 and 1.37 respectively. However, risk has a concomitant relationship with return: the higher the return, the higher the risk.

Indeed, the portfolio [PF_4] has a maximum drawdown of 30% and a daily value-at-risk of about 3.5%. These two values are not negligible, but can be tolerated for a cumulative return of 289%. We conclude that factor analysis can represent an optimal strategy for ambitious investors. Of course, we neglect the other portfolios from the factor analysis ([PF_1], [PF_2] and [PF_3]) because of their poor performance.

On the other hand, the optimal strategy for a risk-averse investor is the numerical optimization approach of the objective function. Indeed, the portfolio [PF_{opt}] has a daily value-at-risk of only 1% and a maximum drawdown of about 13.5%. We can, therefore, deduce that this portfolio is low risk. Moreover, the annual return generated by this portfolio reaches 9% with a cumulative return of 71.5%. Admittedly, the values of these indicators are lower than those calculated on the basis of the factorial approach. However, we cannot deny that the numerical optimisation approach ensures less risk. This approach has therefore created a low-risk portfolio with a significant potential profitability.

In addition, we also have the naive allocation strategy [PF_{eq}], which assigns equal weight to all assets. This approach shows results close to those of numerical optimisation. However, despite the simplicity of this approach, it still performs quite well.

- The analysis of the different strategies leads us to retain 3 main approaches:

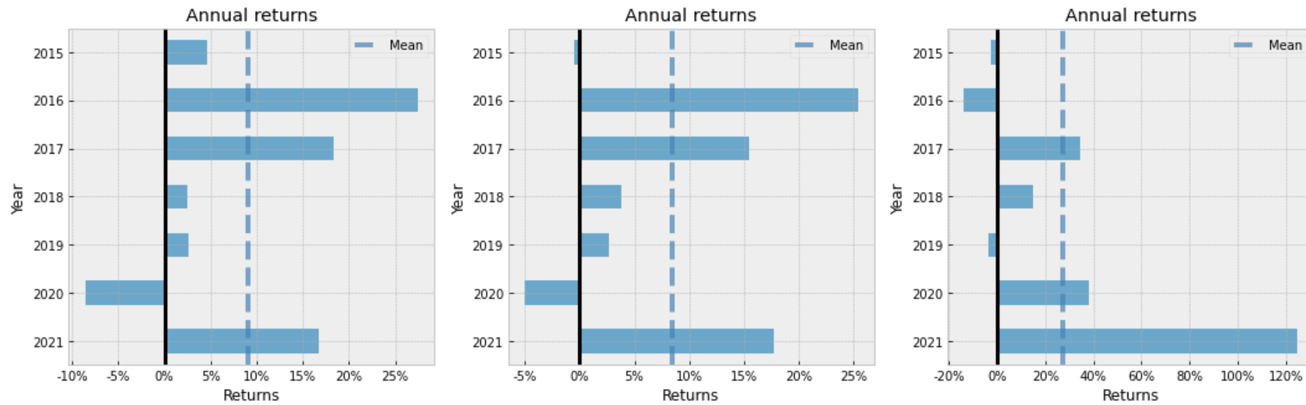


Fig. (4). Annual return on the 3 respective portfolios PF_{eq}, PF_{opt} and PF_4.

- Factorial analysis [PF_4] for the ambitious investor,
- Numerical optimisation [PF_{opt}] for a risk-averse investor,
- The naive approach [PF_{eq}] for a risk-averse and simplistic investor.

Finally, we will analyse the annual profitability of the 3 selected portfolios [PF_{eq}], [PF_{opt}] and [PF_4]:

We find that there is similar annual profitability between the naive approach and the optimisation approach, and that both generate more profitability in an environment characterised by a high level of stability (2016, 2017 and 2018). However, both approaches are not robust enough to adapt to periods of crisis such as the Covid-19 crisis in March 2020. Conversely, the factorial approach, which focuses more on current data than on past data, is highly adaptable to the crisis thanks to its dynamic analysis of the current environment. The conclusion drawn from this analysis is thus confirmed by a monthly analysis of profitability (Annex **A**).

5. CONCLUSION

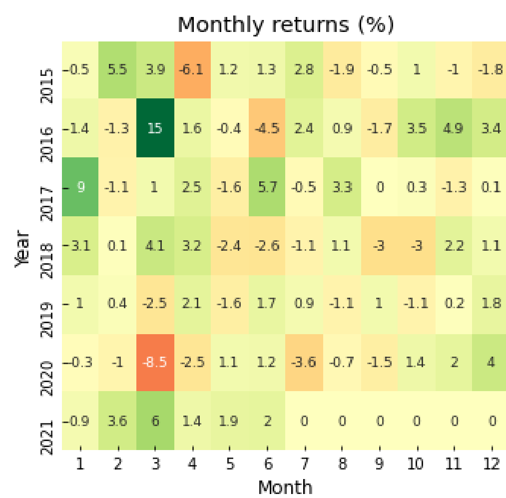
This paper compares several approaches to constructing an optimal portfolio. The results show that the construction of the efficient frontier by simulation does not lead to a satisfactory result in terms of the risk-return trade-off. Optimisation seems to be more efficient in terms of asset allocation, as the portfolio created by numerical optimisation is low risk. Furthermore, we have shown that a naive strategy can also lead to a result close to that of the optimisation. Since this strategy does not require prior knowledge by investors. It can, in some cases, outperform the other approaches.

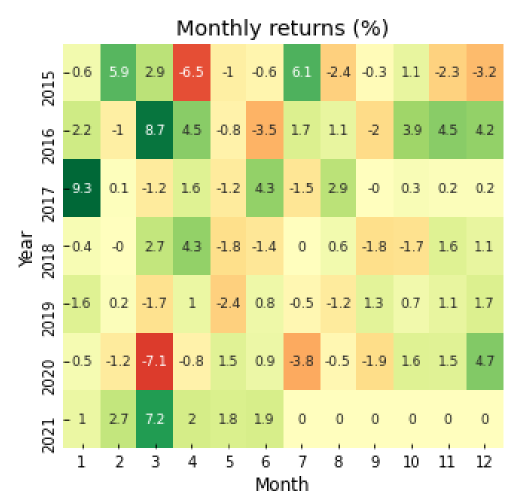
We also used the factorial approach, which is based on the principle components analysis (PCA), in order to identify the data structure and the relationships between variables. In this approach, we used PCA to extract the principal components and overcome the problems of collinearity and redundancy of information. The components are then processed by AIT to obtain independent components that can be used, after normalisation, as proportions in the construction of the 4 portfolios. The results of this approach show that one of the four portfolios created outperforms the other approaches (naive, simulation and optimisation) in terms of profitability while generating a bearable risk. However, the results obtained in this work are based only on historical data of stock prices, but stock prices can be impacted, in the time, by several phenomena (like political, economic, and financial events) that can intervene.

CONFLICT OF INTEREST STATEMENT

The authors declare that they have no conflict of interest.

APPENDICES





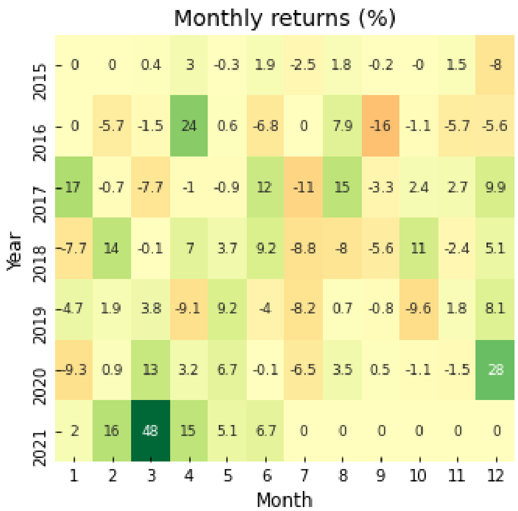


Fig. (5). Monthly return on the 3 respective portfolios PF_{eq}, PF_{opt} et PF_4.

REFERENCES

Billette, Marc-Olivier. « Analyse en composantes indépendantes avec une matrice de mélange éparse », 2013, 91.

Cherfi, Zohra L, Latifa Oukhellou, Patrice Aknin, et Thierry Denœux. « Analyse en composantes indépendantes parcimonieuse pour le diagnostic de systèmes répartis », 2007, 4.

Chitroub, Salim. « Analyse En Composantes Indépendantes D'images Multibandes : Faisabilité Et Perspectives », 2007, 15.

DeMiguel, Victor, Lorenzo Garlappi, et Raman Uppal. « Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? » The review of Financial studies 22, nº 5 (2009): 1915-53.

Fokou, Raoul. « Mesure du risque de marché d'un portefeuille de type Actions - Value-At-Risk, Value-At-Risk Conditionnelle », 2006, 84.

James, Alexander, Yaser S. Abu-Mostafa, et Xiao Qiao. 2019. « Machine Learning for Recession Prediction and Dynamic Asset Allocation ». *The Journal of Financial Data Science* 1 (3): 41-56.

Lei, Ding. 2019. « Black–Litterman asset allocation model based on principal component analysis (PCA) under uncertainty ». *Cluster Computing* 22 (2): 4299-4306.

Markowitz, H. « Portfolio Selection: Efficient Diversification of Investments, Wiley, New York, New York », 1959.

Snow, Derek. 2020. « Machine learning in asset management—part 2: Portfolio construction—weight optimization ». *The Journal of Financial Data Science* 2 (2): 17-24.

Varian, Hal. « A portfolio of Nobel laureates: Markowitz, Miller and Sharpe ». Journal of Economic Perspectives 7, nº 1 (1993): 159-69.

Yang, Libin. « An Application of Principal Component Analysis to Stock Portfolio Management », 2015, 166.

Received: Oct 02, 2021 Revised: Oct 13, 2021 Accepted: Feb 22, 2022